CONTROL SYSTEM DESIGN FOR AN UNMANNED, UNTETHERED, UNDERWATER VEHICLE

by

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ABSTRACT

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ABSTRACT

A procedure for the control system design for a small unmanned, untethered, underwater vehicle was specified. The early phases of this design procedure were performed for the vehicle. The linear equations of motion for small perturbations about an equilibrium condition were developed, and the values of the hydrodynamic coefficients were determined. After the transfer functions were developed to relate the vehicle motion in each of the six degrees of freedom to small deflections of the rudder or stern planes, computer programs were written to determine the poles and zeros of the transfer functions. The vehicle s response in the time domain to one degree deflections of the rudder and stern planes was determined and plotted. The vehicle's response indicated that the vehicle was stable and that the size of the control surfaces could be reduced to minimize drag while maintaining vehicle stability. The nonlinear equations representing the vehicle's motion in six degrees of freedom were incorporated into a computer model of the vehicle. The outputs of this model (linear and angular velocities in six degrees of freedom) are the state variables which will be needed during later phases of the control system design. modern control theory.

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Chapter I

INTRODUCTION

The potential uses for a small unmanned, untethered submersible vehicle (SUUV) are numerous. One of the key systems in the design of such a vehicle is the control system, which must be able to control the vehicle:

1. during transit to and from the vehicle's operating location and depth;
2. while the SUUV performs its mission on location; and
3. during some emergencies for which pre-programmed corrective action has been specified.

In order to perform these functions, the SUUV's control system must be able to accept pre-programmed information concerning course, distance and operating depths as well as real time inputs from various onboard sensors. The output from the control system is used to control the rudder and horizontal control surfaces and ballast control system to cause the SUUV to carry out its programmed assignments.

The length of the mission which the SUUV will be able to perform will be dependent on the amount of energy which can be installed and the rate at which the available energy is consumed. Therefore, the control system should be optimized such that the SUUV expends the smallest amount of energy

while it performs its mission. Designing the control system to minimize the overall vehicle consumption of energy requires that different solutions to the control system design problem be evaluated in terms of energy consumption. For example, the size of the vertical and horizontal control surfaces could be made large enough to provide straight line stability of the vehicle with no control surface deflection. Alternatively, the size of the control surfaces could be made smaller to reduce the drag and thereby reduce the energy required for propulsion. However, smaller control surfaces would require an active control system (an energy drain) to provide vehicle straight line stability.

The design of the control system should follow a logical sequence. Although the demarcations between different phases of the design process are not distinct, the following phases are identified for this project.

Phase I Identify the vehicle size, shape and mission requirements.

Phase II Identify a mathematical model to represent the motion of the vehicle in six degrees of freedom. Linearize the mathematical model.

Phase III Calculate the values of the hydrodynamic coefficients in the linearized equations.

Phase IV Form the transfer functions for each of the linearized equations. Check for vehicle stability by calculating the values of the poles and zeros for each transfer function. Determine the vehicle time response to a small control surface deflection.

Phase V Using modern control theory, model the vehicle in state space using the six nonlinear equations which describe its motion. Include in the model the effects of random environmental disturbances.

Phase VI Model the outputs of the vehicle sensors which will be used to determine the control signal. Include the effects of random noise in the measurements.

Phase VII Construct an observer (Kalman filter) to estimate values of the states which cannot be measured directly by the vehicle's sensors.

Phase VIII Combine Phases V, VI, and VII into a complete model of the vehicle's motion. Use this model to design a linear control system which will control the vehicle in the predicted manner.

In this thesis, Phases I through V are performed in detail, and a brief discussion of Phases VI, VII and VIII is provided.

Phases I and II are discussed in Chapter Two. Figure 1.1 shows the principal dimensions of the SUUV. Although the shape and dimensions were selected arbitrarily, it was felt that these parameters would be close to those which might be selected for an actual vehicle design. The control system design procedure used in this thesis should be appropriate for any vehicle with missions similar to those of the SUUV. Figure 1.2 is a block diagram representation of the anticipated inputs and outputs for the SUUV's control system. The linearized equations which are developed in Chapter Two are adequate to predict the vehicle's response to small perturbations about the assumed equilibrium condition of straight ahead motion with the control surfaces undeflected.

Phase III, calculation of the hydrodynamic coefficients, is performed in Chapter Three.

In Chapter Four, the linearized, small perturbation equations of motion for the vehicle which were developed in Chapter Two are solved individually to find the transfer functions (Phase IV). These transfer functions relate u (forward velocity), w (vertical velocity), and θ (pitch angle) to small deflections of the stern planes; and v (sideslip velocity), ϕ (roll angle), and ψ (yaw angle) to

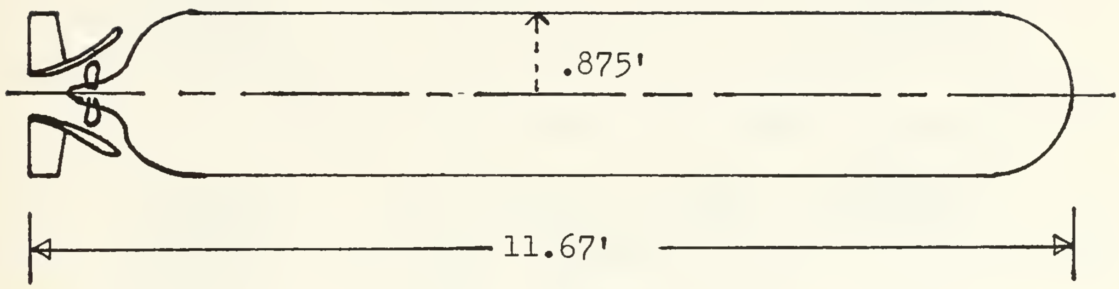

FIGURE 1.1 VEHICLE SHAPE AND DIMENSIONS

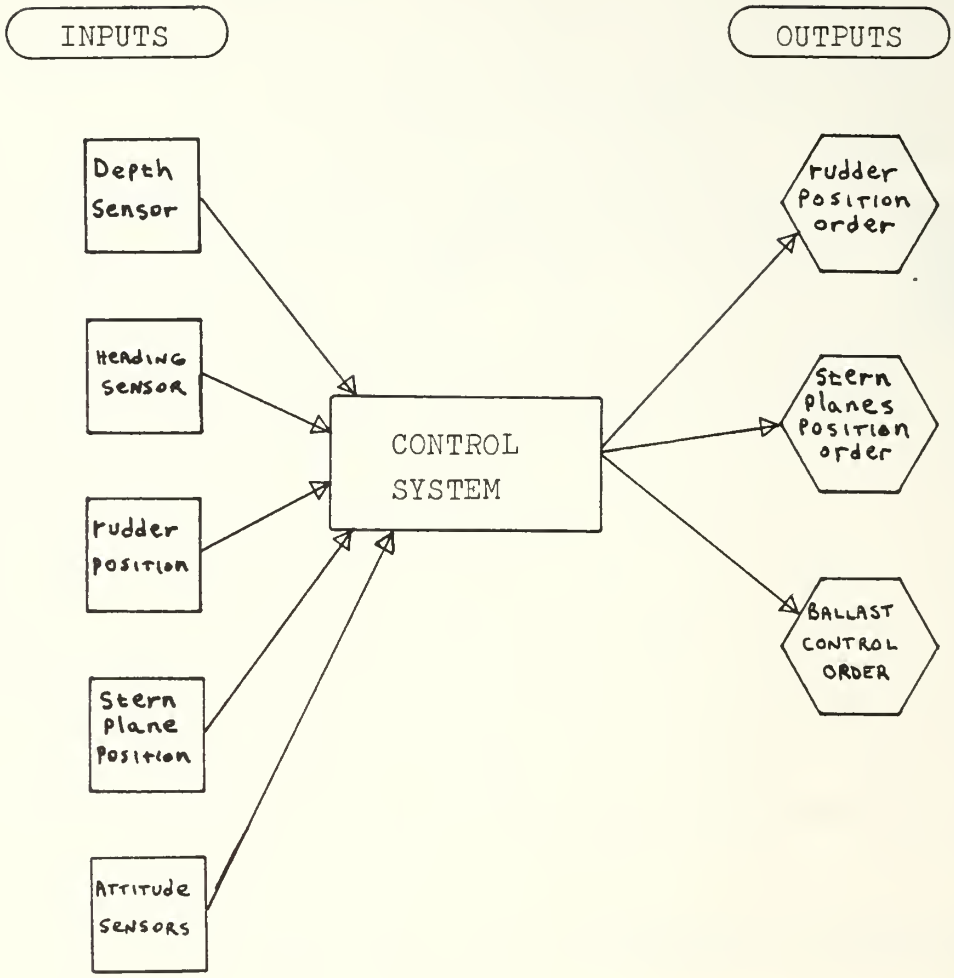

FIGURE 1.2 CONTROL SYSTEM INPUTS AND OUTPUTS

small deflections of the rudder. The poles and zeros of these transfer functions were calculated. Appendix C contains the computer programs used to calculate the poles and zeros of the transfer functions. The inverse LaPlace transforms of the transfer functions were evaluated at a series of times to determine the response of the vehicle to a small step change in rudder or stern plane position. These time response curves were evaluated to determine if the settling time and stability of each response mode were satisfactory. The computer programs used to calculate the time responses are listed in Appendix D.

Having determined from the classical control theory that the initial estimate of the geometry of the vehicle was satisfactory from the aspect of stability and time response, in Chapter Five modern control theory was used to proceed with the beginning of an in-depth design of the control system (Phase V). Figure 1.3 shows a block diagram of the major elements of a modern control system. This thesis includes a development of the model used to represent the motion of the vehicle. This model is a set of nonlinear equations that describe the complete motion of the vehicle in six degrees of freedom - surge, sway, heave, roll, pitch, and yaw - and also includes terms to model the effects of random environmental disturbances on the vehicle. A computer program is described which will evaluate the states (velocities and angular velocities) at a given time.

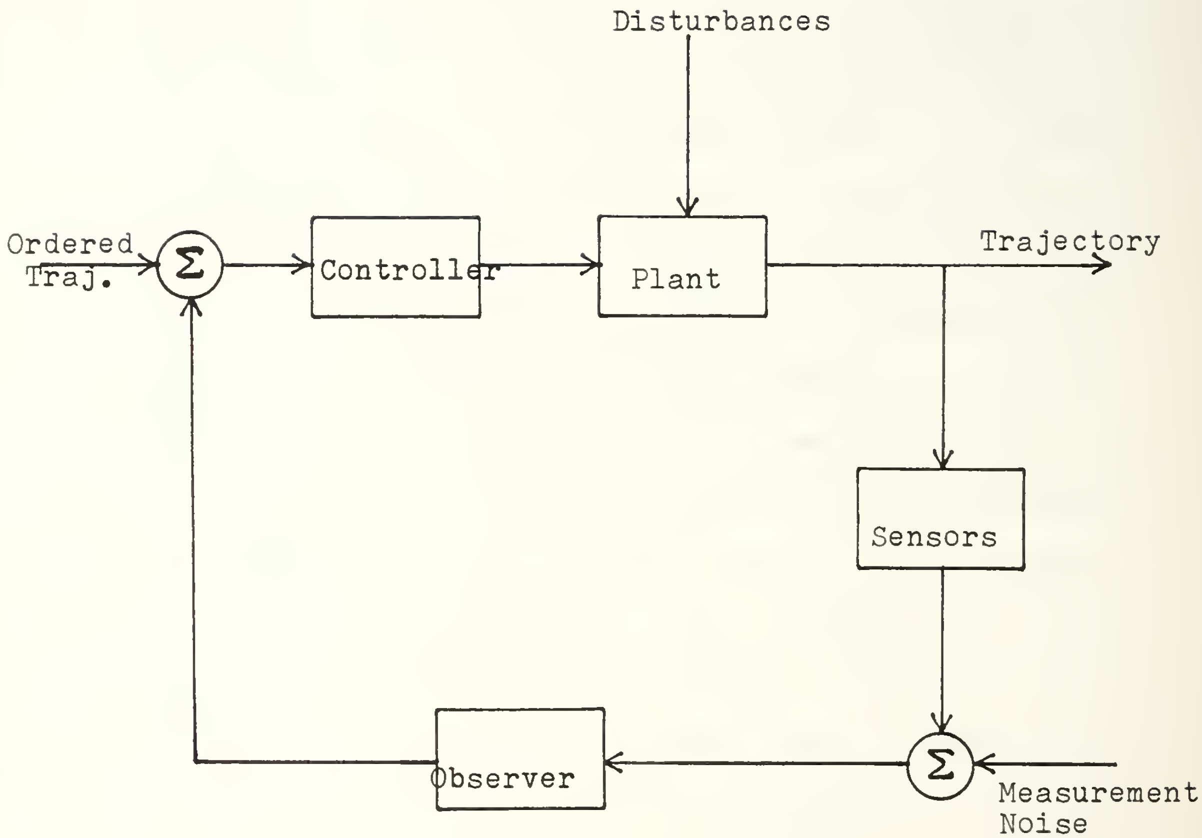

FIGURE 1.3 BLOCK DIAGRAM OF MODERN CONTROL SYSTEM

This program is listed in Appendix E. Follow-on work to this thesis should include developing computer programs to calculate the information discussed in phases VI and VII, and then using the program in Appendix E along with these new programs to model the complete system - input, plant, sensors, observer, disturbances, and output. This would allow simulation of the vehicle motion in real time and the vehicle response under various conditions could be evaluated. Based on these responses, the optimal feedback gains and observer gains can be adjusted if necessary to provide the required vehicle response.

Chapter Six consists of a short summary and the conclusions reached by the author.

Chapter II

EQUATIONS OF MOTION

2.1 IDENTIFICATION OF THE VEHICLE AND ITS MISSION

The small unmanned, untethered, underwater vehicle for which this control system design procedure was developed is depicted in Figure 1.1. The principal dimensions and characteristics of this vehicle are:

Length overall	11.67 feet
Maximum diameter	21 inches
weight	1162 pounds
Buoyancy	1162 pounds
Control surfaces	Rudder
	Stern Planes
Symmetry	Port-Starboard
	Deck-Keel

The mission of the SUUV is:

1. Transit to operating location and depth;
2. Operate for a predetermined period performing the intended mission such as monitoring the environment or conducting inspections of underwater equipment; and
3. Return to the predetermined location for recovery.

2.2 DEVELOPMENT OF THE EQUATIONS OF MOTION

The SUUV was treated as a rigid body with deck-keel and port-starboard symmetry whose motion can be described using six degrees of freedom - surge, sway, heave, pitch, roll, and yaw. The control effectors on the SUUV include a rudder, horizontal control surfaces called stern planes, and a propeller at the stern. Mathematical equations which can be used to model the movement of the SUUV in all six degrees of freedom were then developed following the procedure used by Humphreys[1]. Appendix A contains a list of the notations used in these equations and throughout this thesis. These notations follow the standard nomenclature introduced in the Society of Naval Architects and Marine Engineers TMB 1-5.[2]

The equations which describe the forces, F, and moments, M, acting on the SUUV are:

$$\vec{F} = \frac{d}{dt}\left(\overrightarrow{\text{Momentum}}\right) \tag{1}$$

$$\vec{\mathcal{M}} = \frac{d}{dt}\left(\overrightarrow{\text{Angular momentum}}\right) \tag{2}$$

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[1] Humphreys, D.E.; Development of the Equations of Motion and Transfer Functions for Underwater Vehicles pp. 1-15 Naval Coastal Systems Laboratory; Panama City, Florida, July 1976.

[2] The Society of Naval Architects and Marine Engineers, TMB No. 1-5; Nomenclature for Treating the Motions of a Submerged Body Through a Fluid ; April, 1952.

A set of right-hand, orthogonal coordinate axes centered at the center of mass, CG, of the SUUV was used to develop the equations of motion. Positive directions for the axes, linear velocity components, angular velocity components, angles, forces and moments are shown in Figure 2.1.

The inertial forces and moments acting on the vehicle can be written as:

$$\Sigma X = m(\dot{U} + QW - RV) \tag{3}$$

$$\Sigma Y = m(\dot{V} + RU - PW) \tag{4}$$

$$\Sigma Z = m(\dot{W} + PV - QU) \tag{5}$$

$$\Sigma K = \dot{P}I_x - \dot{R}I_{xz} + QR(I_z - I_y) - PQI_{xz} \tag{6}$$

$$\Sigma M = \dot{Q}I_y + PR(I_x - I_z) - R^2 I_{xz} + P^2 I_{xz} \tag{7}$$

$$\Sigma N = \dot{R}I_z - \dot{P}I_{xz} + PQ(I_y - I_x) + QRI_{xz} \tag{8}$$

The total motion of the vehicle can be assumed to consist of two parts: (1) an average motion representative of the equilibrium condition; and (2) a dynamic motion that consists of small perturbations about the average motion. Thus, the instantaneous velocity components at any time can be written:

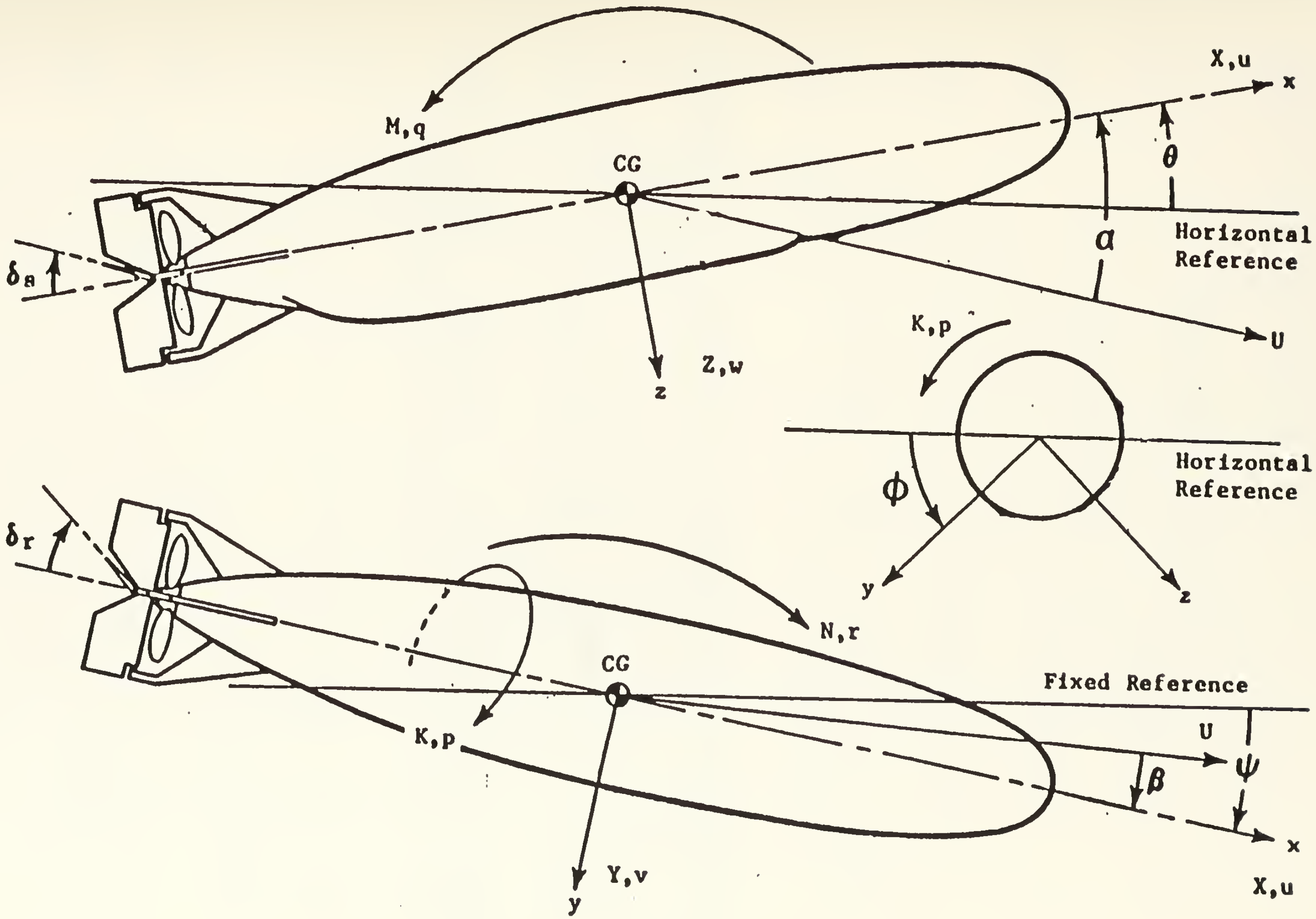


FIGURE 2.1POSITIVE DIRECTIONS OF AXES, ANGLES, VELOCITIES, FORCES AND MOMENTS

$$U = U_0 + u \tag{9}$$

$$V = V_0 + v \tag{10}$$

$$W = W_0 + w \tag{11}$$

$$P = P_0 + p \tag{12}$$

$$Q = Q_0 + q \tag{13}$$

$$R = R_0 + r \tag{14}$$

The zero subscripts represent the average or equilibrium velocity components and the small letters represent the dynamic velocity components. By defining the equilibrium condition as straight ahead motion with control surfaces undeflected, all average velocity components except U_0 are zero and the total velocity equations can be rewritten:

$$U = U_0 + u \tag{15}$$

$$V = v \tag{16}$$

$$W = w \tag{17}$$

$$P = p \tag{18}$$

$$Q = q \tag{19}$$

$$R = r \tag{20}$$

Substituting these equations into the inertial force and moment equations yields:

$$\Sigma X = m(\dot{u} + qw - rv) \tag{21}$$

$$\Sigma Y = m(\dot{v} + rU_0 + ru - pw) \tag{22}$$

$$\Sigma Z = m(\dot{w} + pv - qU_0 - qu) \tag{23}$$

$$\Sigma K = \dot{p}I_x - \dot{r}I_{xz} + qr(I_z - I_y) - pqI_{xz} \tag{24}$$

$$\Sigma M = \dot{q}I_y + pr(I_x - I_z) - r^2 I_{xz} + p^2 I_{xz} \tag{25}$$

$$\Sigma N = \dot{r}I_z - \dot{p}I_{xz} + pq(I_y - I_x) + qrI_{xz} \tag{26}$$

The following assumptions will allow further simplification of these equations.

1. Assume the disturbances from the equilibrium condition are small enough that the products and squares of the changes in velocity are small compared to the changes themselves.
2. Assume the disturbance angles are small enough that the sines of these angles may be set equal to zero and the cosines set equal to one.

3. Products of the disturbance angles are approximately zero.

The following equations result from these assumptions:

$$\Sigma X = m\dot{u} \tag{27}$$

$$\Sigma Y = m(\dot{v} + rU_0) \tag{28}$$

$$\Sigma Z = m(\dot{w} - qU_0) \tag{29}$$

$$\Sigma K = \dot{p}I_x - \dot{r}I_{xz} \tag{30}$$

$$\Sigma M = \dot{q}I_y \tag{31}$$

$$\Sigma N = \dot{r}I_z - \dot{p}I_{xz} \tag{32}$$

Additionally, using the small perturbation assumption and equations (15-20), the relationship between the angular velocities and the rate of change of the angles can be written:

$$p = \dot{\phi} \tag{33}$$

$$q = \dot{\theta} \tag{34}$$

$$r = \dot{\psi} \tag{35}$$

In addition to the inertial forces and moments, the vehicle also experiences hydrodynamic forces and moments due to the forces exerted by the surrounding fluid. These forces and moments are functions of the relative velocity, acceleration and position as well as control surface deflections. They can be expressed in functional form as:

$$F = f(u,v,w,\dot{u},\dot{v},\dot{w},p,q,r,\dot{p},\dot{q},\dot{r},\phi,\theta,\psi,\delta) \tag{36}$$

These forces and moments can be expressed in terms of the Taylor series expansion about the equilibrium condition. Two assumptions allow considerable simplification of these expansions.

1. Assume second order and higher terms may be neglected because only small perturbations from the equilibrium postion are allowed.
2. Assume that since the XZ plane is a plane of symmetry, X , z , and M are functions only of u,w,q,their derivatives, and θ; and Y , N , and K are functions only of v, p, r, their derivatives and ϕ.

Therefore, the hydrodynamic forces and moments expanded in a Taylor series and simplified by the above assumptions can be written:

$$Y_h = Y_o + Y_{\dot{v}}\dot{v} + Y_v v + Y_{\dot{p}}\dot{p} + Y_p p + Y_\phi \phi + Y_r r + Y_{\dot{r}}\dot{r} + Y_\delta \delta \tag{37}$$

$$K_h = K_o + K_{\dot{v}}\dot{v} + K_v v + K_{\dot{p}}\dot{p} + K_p p + K_\phi \phi + K_{\dot{r}}\dot{r} + K_r r + K_\delta \delta \tag{38}$$

$$N_h = N_o + N_{\dot{v}}\dot{v} + N_v v + N_{\dot{p}}\dot{p} + N_p p + N_\phi \phi + N_{\dot{r}}\dot{r} + N_r r + N_\delta \delta \tag{39}$$

$$X_h = X_o + X_{\dot{u}}\dot{u} + X_u u + X_{\dot{w}}\dot{w} + X_w w + X_{\dot{q}}\dot{q} + X_q q + X_\theta \theta + X_\delta \delta \quad (40)$$

$$Z_h = Z_o + Z_{\dot{u}}\dot{u} + Z_u u + Z_{\dot{w}}\dot{w} + Z_w w + Z_{\dot{q}}\dot{q} + Z_q q + Z_\theta \theta + Z_\delta \delta \quad (41)$$

$$M_h = M_o + M_{\dot{u}}\dot{u} + M_u u + M_{\dot{w}}\dot{w} + M_w w + M_{\dot{q}}\dot{q} + M_q q + M_\theta \theta + M_\delta \delta \quad (42)$$

Terms similar to X <u>[3]u expresses the change in the force or moment because of the disturbance velocity. X<u> is called a stability derivative or hydrodynamic coefficient and is defined as the change in the X force with respect to the u velocity and evaluated at the equilibrium condition.

$$X_u = \frac{(\partial X)}{(\partial u)_0}$$

The gravity and buoyancy forces and moments can be expanded in a similar fashion and the complete linearized, small perturbation equations of motion for the SUUV are:

$$m\dot{u} = X_{\dot{u}}\dot{u} + X_u u + X_{\dot{w}}\dot{w} + X_w w + X_{\dot{q}}\dot{q} + X_q q + X_\theta \theta + X_{\delta_s} \delta_s \quad (43)$$

$$m(\dot{w} - U_o\dot{\theta}) = Z_{\dot{u}}\dot{u} + Z_u u + Z_{\dot{w}}\dot{w} + Z_w w + Z_{\dot{q}}\dot{q} + Z_q q + Z_\theta \theta + Z_{\delta_s} \delta_s \quad (44)$$

$$I_y\ddot{\theta} = M_{\dot{u}}\dot{u} + M_u u + M_{\dot{w}}\dot{w} + M_w w + M_{\dot{q}}\dot{q} + M_q q + M_\theta \theta + M_{\delta_s} \delta_s \quad (45)$$

and

---

[3] The notation < > indicates a subscript.

$$m(\dot{v} + U_o\dot{\psi}) = Y_{\dot{v}}\dot{v} + Y_v v + Y_{\dot{p}}\dot{p} + Y_p p + Y_\phi \phi + Y_{\dot{r}}\dot{r} + Y_r r + Y_{\delta_R}\delta_R \quad (46)$$

$$I_x\ddot{\phi} - I_{xz}\ddot{\psi} = K_{\dot{v}}\dot{v} + K_v v + K_{\dot{p}}\dot{p} + K_p p + K_\phi \phi + K_{\dot{r}}\dot{r} + K_r r + K_{\delta_R}\delta_R \quad (47)$$

$$I_z\ddot{\psi} - I_{xz}\ddot{\phi} = N_{\dot{v}}\dot{v} + N_v v + N_{\dot{p}}\dot{p} + N_p p + N_\phi \phi + N_{\dot{r}}\dot{r} + N_r r + N_{\delta_R}\delta_R \quad (48)$$

These two groups of three equations (43-45) and (46-48) are functions of different variables. Hence, the motion of the vehicle can be examined in the vertical (XZ) or horizontal (XY) plane independently of motion in the other plane. (The motion of the vehicle in the two planes is said to be uncoupled.) The first set of equations is usually called the vertical or longitudinal set of equations and the second set is often called the horizontal or lateral set of equations.

Chapter III

CALCULATION OF THE HYDRODYNAMIC COEFFICIENTS

3.1 CONTRIBUTIONS FROM THE BODY

The values for the hydrodynamic coefficients were calculated by first approximating the body shape as an ellipsoid and calculating values for this shape, and then calculating and adding the contributions from the fins and the nozzle. The ellipsoid representing the body is depicted in Figure 3.1.

Volume of Ellipsoid

$$V = 4/3\ \pi\ a\ b^2$$

$$\boxed{V = 18.04\ \text{ft.}^3}$$

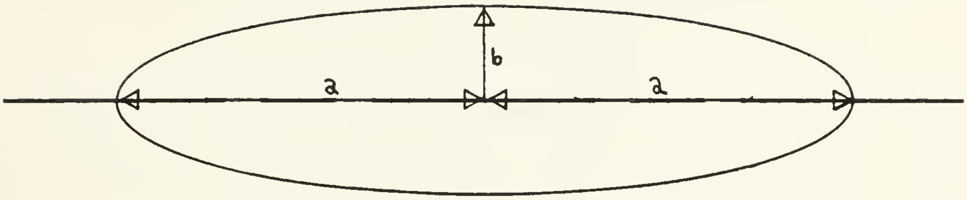
Buoyancy Force of Ellipsoid

$$B = \rho\ V$$

$$\boxed{B = 1162\ \text{lbf.}}$$

Surface Area of Ellipsoid

$$S = 2\pi b^2 + 2\pi \frac{ab}{\epsilon} \sin^{-1}\epsilon$$



$a = 5.625'$

$b = 0.875'$

ϵ = eccentricity

$\epsilon = c/a$

$c = (a^2 - b^2)^{\frac{1}{2}}$

$\epsilon = 0.988$

FIGURE 3.1 ELLIPSOID USED TO REPRESENT VEHICLE

$\epsilon = c/a$ where ϵ = eccentricity[4]

$$\epsilon = .988$$

$$\boxed{S = 49.3 \text{ ft.}^2}$$

Landweber and Johnson[5] determined empirical formulas to determine the values of many of the hydrodynamic coefficients. These formulas contain the terms K',K1,and K2 which are determined for this vehicle by using the table provided by Lamb[6]. Entering this table with the value a/b = 5.714, the values of K', K1, and K2 are:

$$K' = 0.782$$

$$K1 = 0.041$$

$$K2 = 0.925$$

Moments of Inertia

$I_y = I_z$ for a prolate spheroid

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[4] Thomas,G.B.Jr.; Calculus and Analytic Geometry ; p. 478; Addison-Wesley Publishing Co.; Reading, Ma.; 1960.

[5] Landweber, L. and Johnson,J.L.; Prediction of Dynamic Stability Derivatives of an Elongated Body of Revolution ; NSRDC Report C-359; May,1951.

[6] Lamb, Sir Horace; Hydrodynamics, 6th Ed.; pp. 155; Dover Publications,New York, 1945.

$$I_y = m(a^2 + b^2)/\ 5$$

$$\boxed{I_y = 234 \text{ slugs-ft.}^2}$$

$$\boxed{I_z = 234 \text{ slugs-ft.}^2}$$

$$I_y' = I_y/(1/2\rho l^5)$$

$$\boxed{I_y' = I_z' = 0.0013}$$

$$I_x = m(2b^2)\ /\ 5$$

$$\boxed{I_x = 11.1 \text{ slugs-ft.}^2}$$

$$I_x' = I_x/(1/2\rho l^5)$$

$$\dot{I}_x' = .0000614$$

Mass

$m = 1162/32.2$

$$m = 36.1 \text{ slugs}$$

$$m' = m/(1/2\rho l^3)$$

$$m' = .02535$$

X_G, y_G and Z_G

$X_G = Y_G = 0$

$Z_G = -1/2\text{inch} = -.042\text{ft.}$

$$z_G' = -.0037$$

Coefficient of Drag

Newman[7] has determined that for three dimensional bodies where the maximum thickness is less than one-fifth of the length, the frictional drag is dominant and the drag coefficient can be predicted from the flat-plate drag coefficient C<f>. Assuming a speed of five knots, the Reynolds number is:

$$Re = Ul/\nu$$

$$Re = (5)(1.689)(10)(.0929)/(1.35 \times 10^{-6})$$

$$Re = 5.81 \times 10^{6} \quad \text{(for seawater at } 10^{0}\text{C)}$$

From Figure 2.3 in Newman[8]

$$C_f = 3.5 \times 10^{-3}$$

$$\text{and since} \quad C_D = C_f$$

$$C_D = 0.0035$$

$$X_u = \frac{\partial x}{\partial u}$$

$$X = -1/2\rho u^2 A C_D$$

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[7] Newman, J.N.; Marine Hydrodynamics ; pp 20; MIT Press, Cambridge, Ma.; 1977.

[8] ibid

$$\partial X/\partial u = -\rho u A C_D$$

A = S = Surface Area

$$A = 49.3\ l^2 / 126.6 = .39\ l^2$$

$$X_u' = 2\ X_u / \rho u l^2$$

$$\boxed{X_u' = -0.00273}$$

$$X_{\dot{u}}' = -K_1 m'$$

$$\boxed{X_{\dot{u}}' = -0.00104}$$

$\underline{Y}_v$ $\underline{\text{and}}$ $\underline{Z}_w$

$$Y_w' = Z_v'$$

$$Z_w' = -0.234(m')^{0.79} - D'$$

where $D' = C_D S/l^2$ $\quad D' = .00137$

$$\boxed{Z_w' = -.0142}$$

$$\boxed{Y_v' = -.0142}$$

$\underline{Y_{\dot{v}}'}$ and $\underline{Z_{\dot{v}}'}$

$$Y_{\dot{v}}' = Z_{\dot{v}}' \qquad \text{for an ellipsoid}$$

$$Y_{\dot{v}}' = -K_2 m'$$

$$\boxed{Y_{\dot{v}}' = -\ .02345}$$

$$\boxed{Z_{\dot{w}}' = -\ .02345}$$

$\underline{M_w}$ and $\underline{N_v}$

$$M_w' = 0.87(K_2 - K_1)m'$$

$$\boxed{M_w' = .0195}$$

$$\boxed{N_v' = -.0195}$$

$\underline{Z_q}$ and $\underline{Y_r}$

$$Z_q' = -(0.10 - K_1)m'$$

$$\boxed{Z_q' = -.0015}$$

$$\boxed{Y_r' = .0015}$$

$\underline{M_q}$ $\underline{\text{and}}$ $\underline{N_r}$

$$M_q' = -0.045\ m'$$

$$\boxed{M_q' = -.00114}$$

$$\boxed{N_r' = -.00114}$$

$\underline{M_{\dot{q}}}$ $\underline{\text{and}}$ $\underline{N_{\dot{r}}}$

$$M_{\dot{q}}' = -K'I_y'$$

$$\boxed{M_{\dot{q}}' = -.00102}$$

$$\boxed{N_{\dot{r}}' = -.00102}$$

$\underline{M_\theta}$ and $\underline{K_\phi}$

$$M_\theta' = m'g\,Z_G'$$

$$\boxed{M_\theta' = -.00302}$$

$$\boxed{K_\phi' = -.00302}$$

The circular symmetry of an ellipsoid causes the following coefficients to be zero:

$$\boxed{K_{\dot p}' = 0 \quad Z_{\dot q}' = 0 \quad M_{\dot w}' = 0 \quad Y_{\dot r}' = 0 \quad N_{\dot v}' = 0}$$

3.2 CONTRIBUTIONS FROM THE FINS

The size and shape of the fins selected was the same for the rudder and the stern planes. Figure 3.2 shows the dimensions and shape of one of these fins. The total area of both the port and starboard stern planes was calculated to be 0.4 square feet.

$$\boxed{A = 0.4 \text{ ft.}^2}$$

The average span of the rudder and stern planes was calculated as follows:

$$S = ((.578 + .375) / (2))2$$

$$S = .972 \text{ ft.}$$

The average chord was calculated to be:

$$C = .365 \text{ ft.}$$

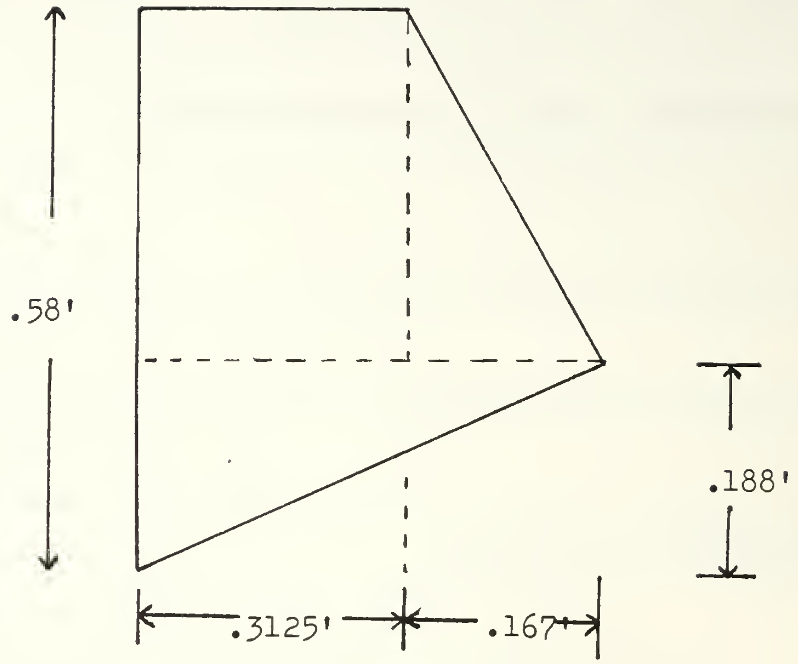
Aspect Ratio

$$AR = S/C$$

$$AR = 2.66$$

$$Z'_w = -.5\rho U_0 A\, C_{f\,L\alpha} / (.5\rho U_0 l_2)$$

$$C_{L\alpha} = 2\pi/(1+2/AR)$$



AREA = .033 ft.^{2}

AREA = .045 ft.^{2}

AREA = .122 ft.^{2}

Total Area (1 fin) = .2 ft.^{2}

Total Area (2 fins) = .4 ft.^{2}

FIGURE 3.2 FIN SIZE AND SHAPE

$$C_{L\alpha} = 3.59$$

For calculating the contributions of the fins to the hydrodynamic coefficients N<v>, M<w>, N<r>, N<v>, y<r>, y<v>, Z<w>, Z<q>, and M<q>, the projected area of the nozzle also was included in the calculation of effective fin area and span. Figure 3.3 shows the effective fin used in these calculations.

$$Z'_w = -A_f C_{L\alpha} / l^2$$

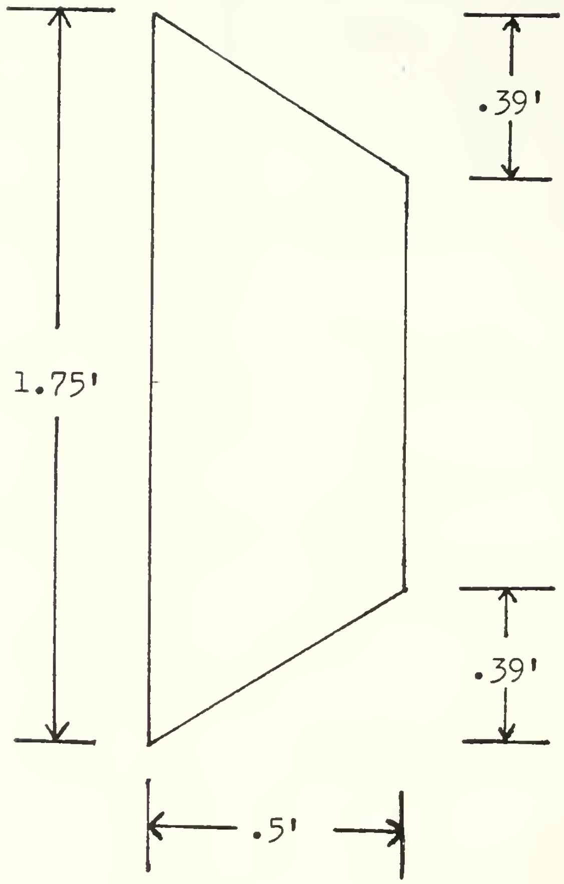
$$\boxed{Z'_w = -.040184}$$

$$Z'_{\dot{w}} = -8\pi S^2C^2 /((4\sqrt{S^2 + C^2})l^3)$$

$$\boxed{Z'_{\dot{w}} = -.000275}$$

$$Z'_{\dot{q}} = (l_x / l)\cdot(Z'_{\dot{w}})$$

$$\boxed{Z'_{\dot{q}} = -.00014}$$



Area = .81 ft.^{2}

Span ($\bar{S}$) = 1.75'

Chord ($\bar{C}$) = .48'

Aspect Ratio (AR) $\quad AR = \frac{\bar{S}}{\bar{C}} \quad AR = 3.65$

$$C_L = \frac{2\pi}{1 + \frac{2}{AR}}$$

FIGURE 3.3 EFFECTIVE FIN AREA

$$Y.'_{r} = -Z.'_{w}(l_{x} / l)$$

$$\boxed{Y.'_{r} = .00014}$$

$$Y'_{v} = Z'_{w}$$

$$\boxed{Y'_{v} = -.02598}$$

$$Y.'_{v} = Z.'_{w}$$

$$\boxed{Y.'_{v} = -.000275}$$

$$Z'_{q} = (l_{x}/l)\bullet(Z'_{w})$$

$$\boxed{z'_{q} = -\ .01461}$$

$$Y_r' = -Z_q'$$

$$\boxed{Y_r' = .01461}$$

$$K_p' = (l_y^2/l)(Z_w')$$

$$\boxed{K_p' = -.0000445}$$

$$K_{\dot{p}}' = (l_y^2/l)(Z_{\dot{w}}')$$

$$\boxed{K_{\dot{p}}' = -.0000017}$$

$$M_w' = (l_x/l)(Z_w')$$

$$\boxed{M_w' = -.01311}$$

$$N_v' = M_w'$$

$$\boxed{N_{v}' = .001311}$$

$$M_{\dot{w}}' = (l_x/l)(Z_{w}')$$

$$\boxed{M_{\dot{w}}' = -\ .00014}$$

$$-N_{\dot{v}}' = M_{\dot{w}}'$$

$$\boxed{N_{\dot{v}}' = \ .00014}$$

$$M_{q}' = (l_x^{\,2}/l)(Z_{w}')$$

$$\boxed{M_{q}' = -.00775}$$

$$M_{\dot{q}}' = (l_x^{\,2}/l)(Z_{\dot{w}}')$$

$$\boxed{M_{\dot{q}}' = -.0000017}$$

$$N_r' = M_q'$$

$$\boxed{N_r' = -.00775}$$

$$N_{\dot{r}}' = M_{\dot{q}}'$$

$$\boxed{N_{\dot{r}}' = -\ .0000017}$$

$$Y_{\delta R} = C_{L\alpha} A_f / l^2$$

$$\boxed{Y_{\delta R} = .0074}$$

$$Z_{\delta S} = -Y_{\delta R}$$

$$Z_{\delta S} = -.0074$$

$$K_{\delta R} = 0.0$$

$$M_{\delta S} = -l_x C_{L\alpha} A_f / (l^3)$$

$$M_{\delta S} = -.0037$$

$$N_{\delta R} = M_{\delta S}$$

$$N_{\delta R} = -.0037$$

Table 3-1 lists the values of all of the hydrodynamic coefficients for ease of reference.

TABLE 3-1 NONDIMENSIONAL VALUES OF HYDRODYNAMIC DERIVATIVES

Hydrodynamic Derivative	BODY	FINS	TOTAL
X_u	-0.00273		-0.00273
$X_{\dot{u}}$	-0.00104		-0.00104
Y_v	-0.01420	-0.025980	-0.04018
$Y_{\dot{v}}$	-0.02345	-0.000275	-0.02373
Y_r	0.00150	0.01310	0.0146
$Y_{\dot{r}}$	0.0	-0.000140	-0.00014
Z_w	-0.01420	-0.02598	-0.04018
$Z_{\dot{w}}$	-0.02345	-0.000275	-0.02373
Z_q	-0.00150	-0.01310	-0.01460
$Z_{\dot{q}}$	0.0	-0.000140	-0.00014
K_ϕ	-0.00302		-0.00302
K_p	0.0	-0.0000445	-0.0000445
$K_{\dot{p}}$	0.0	-0.0000017	-0.0000017
M_w	0.01950	-0.01311	0.006395
$M_{\dot{w}}$	0.0	-0.000140	-0.00014

Table 3-1 (Con't)

Hydrodynamic Derivative	BODY	FINS	TOTAL
M_{θ}	-0.00302		-0.00302
M_{q}	-0.00114	-0.006610	-0.00775
$M_{\dot{q}}$	-0.00102	-0.0000017	-0.00102
N_{v}	-0.01950	0.001311	-0.01819
$N_{\dot{v}}$	0.0	0.000140	0.000140
N_{r}	-0.00114	-0.006610	-0.00775
$N_{\dot{r}}$	-0.00102	-0.0000017	-0.00102
$Y_{\delta R}$		0.007400	0.00740
$Z_{\delta S}$		-0.007400	-0.00740
$K_{\delta R}$		0.0	0.0
$M_{\delta S}$		-0.003700	-0.00370
$N_{\delta R}$		-0.003700	-0.00370

Chapter IV

DEVELOPMENT OF THE TRANSFER FUNCTIONS

4.1 GENERAL

The linearized, small perturbation equations of motion were developed in Chapter Two. After substituting equations 33, 34, and 35 into the equations of motion, the longitudinal and lateral equations of motion can be written:

LONGITUDINAL EQUATIONS

$$m\dot{u} - X_{\dot{u}}\dot{u} - X_u u - X_{\dot{w}}\dot{w} - X_w w - X_{\dot{q}}\ddot{\theta} - X_q\dot{\theta} - X_\theta\theta = X_{\delta_s}\delta_s \tag{49}$$

$$m\dot{w} - mU_o\dot{\theta} - Z_{\dot{u}}\dot{u} - Z_u u - Z_{\dot{w}}\dot{w} - Z_w w - Z_{\dot{q}}\ddot{\theta} - Z_q\dot{\theta} - Z_\theta\theta = Z_{\delta_s}\delta_s \tag{50}$$

$$I_y\ddot{\theta} - M_{\dot{u}}\dot{u} - M_u u - M_{\dot{w}}\dot{w} - M_w w - M_{\dot{q}}\ddot{\theta} - M_q\dot{\theta} - M_\theta\theta = M_{\delta_s}\delta_s \tag{51}$$

LATERAL EQUATIONS

$$m\dot{v} + mU_o\dot{\psi} - Y_{\dot{v}}\dot{v} - Y_v v - Y_{\dot{p}}\ddot{\phi} - Y_p\dot{\phi} - Y_\phi\phi - Y_{\dot{r}}\ddot{\Psi} - Y_r\dot{\psi} = Y_{\delta_R}\delta_R \tag{52}$$

$$I_x\ddot{\phi} - I_{xz}\ddot{\Psi} - K_{\dot{v}}\dot{v} - K_v v - K_{\dot{p}}\ddot{\phi} - K_p\dot{\phi} - K_\phi\phi - K_{\dot{r}}\ddot{\Psi} - K_r\dot{\psi} = K_{\delta_R}\delta_R \tag{53}$$

$$I_z\ddot{\Psi} - I_{xz}\ddot{\phi} - N_{\dot{v}}\dot{v} - N_v v - N_{\dot{p}}\ddot{\phi} - N_p\dot{\phi} - N_\phi\phi - N_{\dot{r}}\ddot{\Psi} - N_r\dot{\psi} = N_{\delta_R}\delta_R \tag{54}$$

The terms X<δs>δs, Z<δs>δs, M<δs>δs, Y<δR>dR, K<δR>δR, and N<δR>δR are the control forces and moments generated when the control system causes small deflections of the

stern planes or rudder. The transfer functions for the response of the vehicle to small deflections of the rudder or stern planes are developed in the following pages.

4.2 LONGITUDINAL TRANSFER FUNCTIONS

Taking the LaPlace transform of the longitudinal equations yields:

$$[(m - X_{\dot{u}})s - X_u]u + [- X_{\dot{w}}s - X_w]w + [- X_{\dot{q}}s^2 - X_q s - X_\theta]\theta = X_{\delta_s}\delta_s \tag{55}$$

$$[- Z_{\dot{u}}s - Z_u]u + [(m - Z_{\dot{w}})s - Z_w]w + [- Z_{\dot{q}}s^2 - (Z_q + mU_o)s - Z_\theta]\theta = Z_{\delta_s}\delta_s \tag{56}$$

$$[- M_{\dot{u}}s - M_u]u + [- M_{\dot{w}}s - M_w]w + [(I_y - M_{\dot{q}})s^2 - M_q s - M_\theta]\theta = M_{\delta_s}\delta_s \tag{57}$$

These equations were nondimensionalized by dividing the force equations by $1/2\ \rho l^2 U_0{}^2$, the moment equations by $1/2\ \rho l^3 U_0{}^2$, and S by U_0/l. Then the nondimensionalized equations can be written:

NONDIMENSIONAL LONGITUDINAL EQUATIONS OF MOTION

$$[(m' - X'_{\dot{u}})s' - X'_u]u' - [X'_{\dot{w}}s' + X'_w]w' - [X'_{\dot{q}}s'^2 + X'_q s' + X'_\theta]\theta = X'_{\delta_s}\delta_s \tag{58}$$

$$\begin{aligned}[-Z'_{\dot{u}}s' - Z'_u]u' &+ [(m' - Z'_{\dot{w}})s' - Z'_w]w' \\ &- [Z'_{\dot{q}}s'^2 + (Z'_q + m')s' + Z'_\theta]\theta = Z'_{\delta_s}\delta_s\end{aligned} \tag{59}$$

$$[- M'_{\dot{u}}s' - M'_u]u' - [M'_{\dot{w}}s' + M'_w]w' + [(I'_y - M'_{\dot{q}})s'^2 - M'_q s' - M'_\theta]\theta = M'_{\delta_s}\delta_s \tag{60}$$

The transfer functions can be determined for a given input by solving the transformed simultaneous equations of motion for the variable of interest with all other inputs set equal to zero. For example, the pitch attitude to stern plane deflection angle transfer function, using determinants, can be written:

$$\frac{\theta(s)}{\delta_s(s)} = \frac{\begin{vmatrix} (m' - X'_{\dot u})s' - X'_u & -X'_{\dot w}s' - X'_w & X'_{\delta_s} \\ -Z'_{\dot u}s' - Z'_u & (m' - Z'_{\dot w})s' - Z'_w & Z'_{\delta_s} \\ -M'_{\dot u}s' - M'_u & -M'_w s' - M'_w & M'_{\delta_s} \end{vmatrix}}{\begin{vmatrix} (m' - X'_{\dot u})s' - X'_u & -X'_{\dot w}s' - X'_w & -X'_{\dot q}s'^2 - X'_q s' - X'_\theta) \\ -Z'_{\dot u}s' - Z'_u & (m' - Z'_{\dot w})s' - Z'_w & -Z_{\dot q}s'^2 - (Z'_q + m')s' + -Z'_\theta \\ -M'_{\dot u}s' - M'_u & -M'_{\dot w}s' - M'_w & (I'_y - M'_{\dot q})s'^2 - M'_q s' - M'_\theta \end{vmatrix}}$$

These determinants were expanded, resulting in a numerator polynomial in s' over a denominator polynomial in s'. The denominator polynomial for the three longitudinal transfer functions is a common polynomial. Setting this denominator equal to zero gives the characteristic equation whose roots are equal to the poles of the system. The damping and the natural frequency and the time constant of the system can be determined from this characteristic equation. The

longitudinal transfer functions can be summarized as follows:

LONGITUDINAL TRANSFER FUNCTIONS

$$\theta/\delta_s = \frac{N^{\theta}_{\delta_s}}{\Delta_{Long}} = \frac{A_\theta s'^2 + B_\theta s' + C_\theta}{As'^4 + Bs'^3 + Cs'^2 + Ds' + E} \tag{61}$$

$$w'/\delta_s = \frac{N^{W}_{\delta_s}}{\Delta_{Long}} = \frac{A_w s'^3 + B_w s'^2 + C_w s' + D_w}{As'^4 + Bs'^3 + Cs'^2 + Ds' + E} \tag{62}$$

$$u'/\delta_s = \frac{N^{U}_{\delta_s}}{\Delta_{Long}} = \frac{A_u s'^3 + B_u s'^2 + C_u s' + D_u}{As'^4 + Bs'^3 + Cs'^2 + Ds' + E} \tag{63}$$

The coefficients A,B,C,D, and E and the coefficients in the numerators are evaluated using the equations in Appendix B. The values of the hydrodynamic derivatives used to evaluate these coefficients are as calculated in Chapter Three. In order to evaluate the transfer functions, a computer program was written which first determined the values of the coefficients using the equations in Appendix B, and then used these values to determine the roots of the numerator and denominator.

Using the program VERTLIN.FORT (listed in Appendix C) the roots of the polynomials in the longitudinal transfer functions were calculated . See Table 4 - 1. The surge (u) polynomial coefficients and roots are zero because all of

```
OUTPUT VALUES ARE IN DIMENSIONAL FORM
COEFFS. OF CHARACTERISTIC EQUATION

      A                   B                   C                   D                   E
 0.974719E-05   0.306286E-04   0.206294E-04   0.568815E-05   0.331301E-06

ROOTS OF CHARACTERISTIC EQUATION

        REAL            IMAGINARY
     -2.342500          0.0

     -0.361070          0.237651

     -0.361070         -0.237651

     -0.077655          0.0

PITCH POLYNOMIAL COEFFICIENTS

      ATHETA          BTHETA          CTHETA
 -0.850454E-05 -0.754724E-05 -0.535090E-06
ROOTS
        REAL            IMAGINARY
     -0.809735          0.0

     -0.077702          0.0

VERTICAL VELOCITY COEFFICIENTS

      AW            BW            CW            DW
 -0.110338E-05 -0.463001E-05 -0.113874E-05 -0.610100E-07
ROOTS
        REAL            IMAGINARY
     -3.937668          0.0

     -0.180908          0.0

     -0.077621          0.0

FORWARD SPEED POLYNOMIAL COEFFICIENTS

      AU              BU              CU              DU
  0.0             0.0             0.0             0.0
ROOTS
        REAL            IMAGINARY
****************        0.0

****************        0.0

****************        0.0
```

TABLE 4-1 ROOTS OF VERTICAL TRANSFER FUNCTION

the hydrodynamic coefficients associated with X were assumed to be zero except X<u> and X<u>. Figure 4.1, which is a flow chart for VERTLIN.FORT shows the method used to calculate the roots. Subroutine ZPOLR, a routine written by IMSL,Inc. and available as a library routine at C.S. Draper Laboratory, was used in VERTLIN.FORT to find the zeros of the polynomials. The pitch (θ)and heave (w) transfer functions can be written (after cancellation):

$$\theta/\delta s = \frac{(S + .81)}{(S + 2.34)(S + .36 + .24j)(S + .36 - .24j)} \tag{64}$$

$$w/\delta s = \frac{(S + 3.94)(S + .18)}{(S + 2.34)(S + .36 + .24j)(S + .36 - .24j)} \tag{65}$$

The vehicle's response in the vertical plane was then determined by finding the inverse LaPlace transform for θ(s) and w(s) assuming a one degree deflection of the stern planes. Pitch angle as a function of time following a one degree deflection can be expressed:

$$\begin{aligned} \theta(t) = & -0.027 - 0.0025(\exp)(-2.34t) \\ & + 0.03(\exp)(-.36t)(\cos.24t + .66\sin.24t) \end{aligned} \tag{66}$$

The vehicle's vertical velocity as a function of time following a one degree deflection of the stern planes can be expressed:

$$\begin{aligned} w(t) = & -.003 + .001(\exp)(-2.34t) \\ & +0.002(\exp)(-.36t)(\cos.24t + .032\sin.24t) \end{aligned} \tag{67}$$

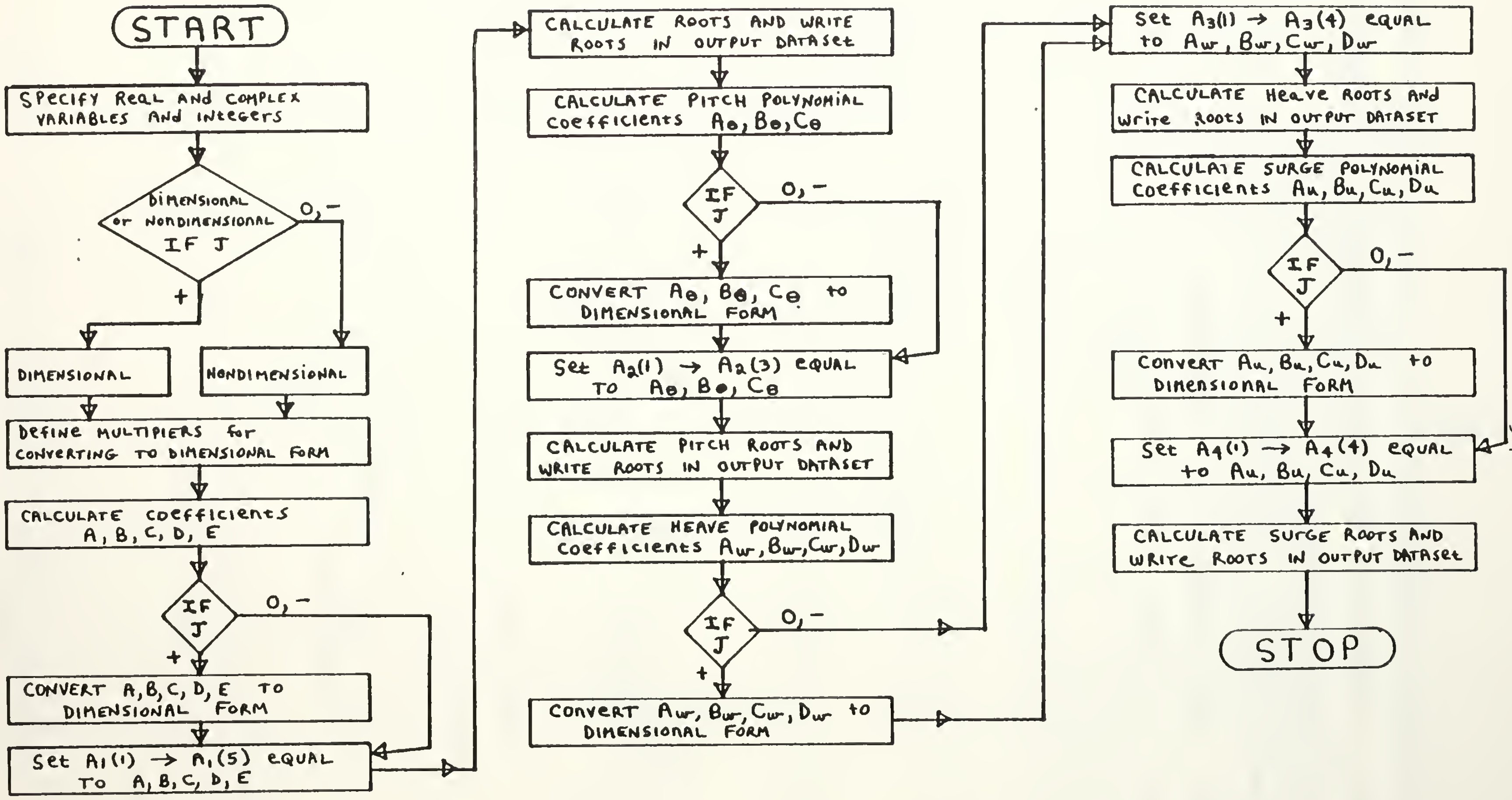


FIGURE 4.1 FLOWCHART FOR VERTLIN.FORT

Plots of $\theta(t)$ and $w(t)$ are shown in Figure 4.2 and 4.3 These plots show that the vehicle behaves in a stable, predictable manner in the vertical plane following a small stern plane deflection.

In the expressions for pitch angle and vertical velocity as functions of time, it can be seen that some terms are much smaller than others. This indicates that the effects of some poles and zeros are insignificant in comparison to the effects from the remaining poles and zeros. When these zeros and poles which have only minor effects are dropped from the transfer functions, the transfer functions can be written:

$$\theta(s) = \frac{(S + .81)}{(S + .36 + .24j)(S + .36 - .24j)} \tag{68}$$

$$w(s) = \frac{(S + .18)}{(S + .36 + .24j)(S + .36 - .24j)} \tag{69}$$

The system characteristic equation can now be written:

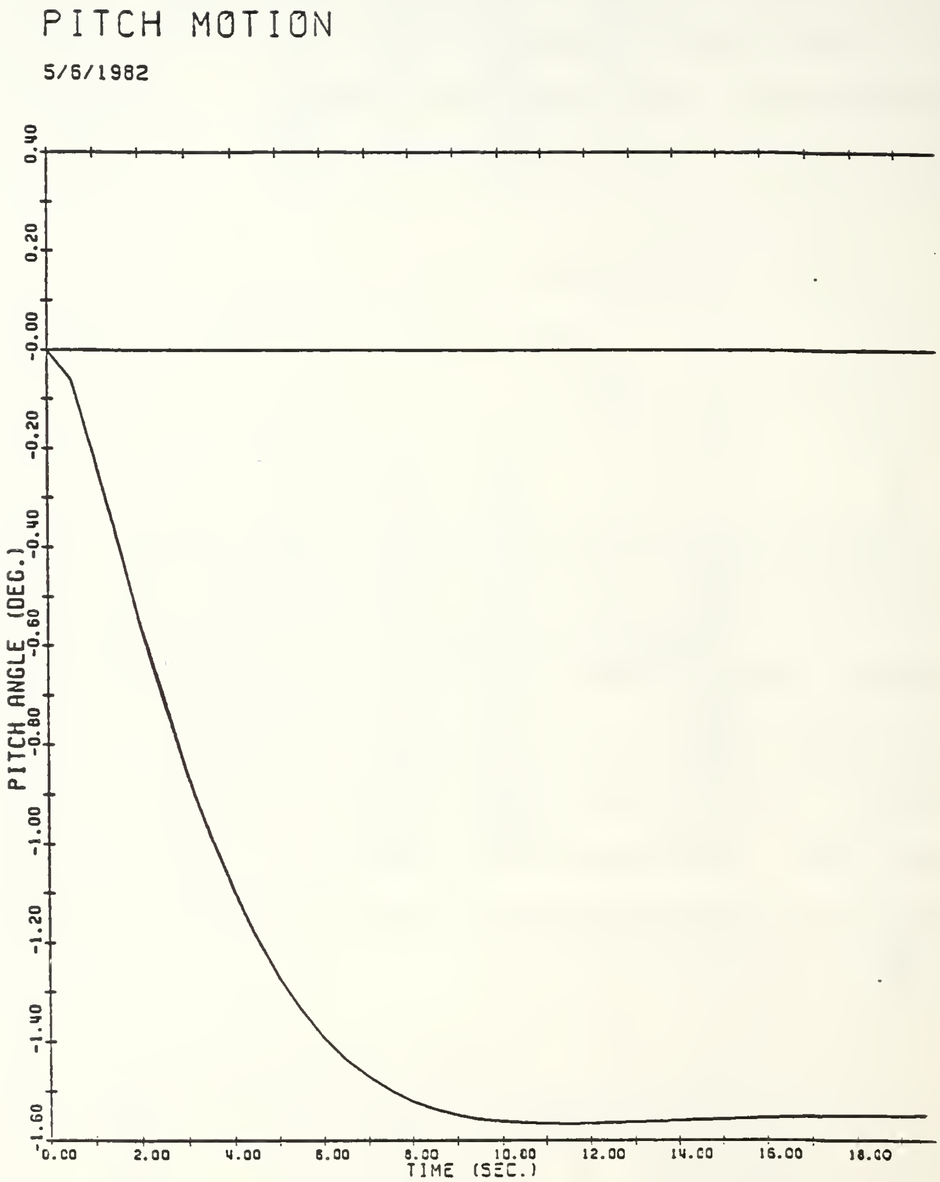


FIGURE 4.2 PITCH MOTION FOR A ONE DEGREE STERN PLANE DEFLECTION

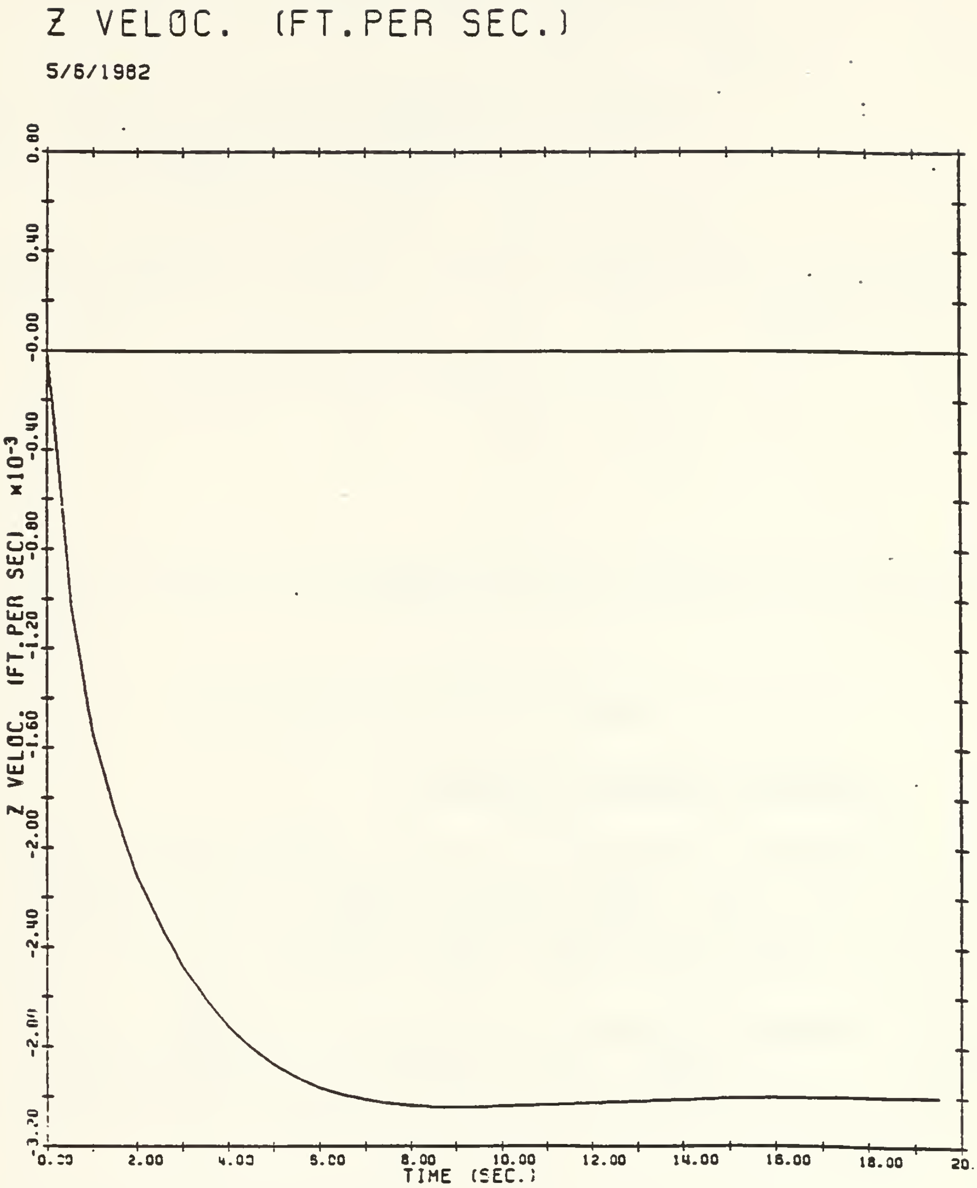


FIGURE 4.3 HEAVE MOTION FOLLOWING A ONE DEGREE STERN PLANE DEFLECTION

$$S^2 + .76S + .187 = 0 \qquad (70)$$

This equation is of the form,

$$S^2 + 2\zeta\omega n + \omega n^2 = 0$$

where ζ = damping ratio

ωn = natural frequency

Solving for the damping ratio, natural frequency and system time constant:

ζ = 0.88

ωn = 0.432

T = 14.5 secs.

4.3 LATERAL TRANSFER FUNCTIONS

The transfer functions for the lateral equations were determined and analyzed using the same procedure just completed for the longitudinal equations. Equations (52-54) were nondimensionalized in the same fashion as were the longitudinal equations, with the additional definition that $\beta^0 = v/U_0$.

Nondimensionalized Lateral Equations of Motion

$$[(m' - Y'_{\dot{v}})s' - Y'_v]\beta + [- Y'_{\dot{p}}s'^2 - Y'_p s' - Y'_\phi]\phi$$
$$+ [- Y'_{\dot{r}}s'^2 + (m' - Y'_r)s']\psi = Y'_{\delta_R}\delta_R \tag{71}$$

$$[- K'_{\dot{v}}s' - K'_v]\beta + [(I'_x - K'_{\dot{p}})s'^2 - K'_p s' - K'_\phi]\phi$$
$$+ [(- I'_{xz} - K'_{\dot{r}})s'^2 - K'_r s']\psi = K'_{\delta_R}\delta_R \tag{72}$$

$$[- N'_{\dot{v}}s' - N'_v]\beta + [(- I'_{xz} - N'_{\dot{p}})s'^2 - N'_p s' - N'_\phi]\phi$$
$$+ [(I'_z - N'_{\dot{r}})s'^2 - N'_r s']\psi = N'_{\delta_R}\delta_R \tag{73}$$

The lateral transfer functions were determined from these equations.

Lateral Transfer Functions

$$\beta/\delta_R = \frac{N^\beta_{\delta_R}}{\Delta_{Lat}} = \frac{s'(A_\beta s'^3 + B_\beta s'^2 + C_\beta s' + D_\beta)}{s'(As'^4 + Bs'^3 + Cs'^2 + Ds' + E)} \tag{74}$$

$$\phi/\delta_R = \frac{N^\phi_{\delta_R}}{\Delta_{Lat}} = \frac{s'(A_\phi s'^2 + B_\phi s' + C_\phi)}{s'(As'^4 + Bs'^3 + Cs'^2 + Ds' + E)} \tag{75}$$

$$\psi/\delta_R = \frac{N^\psi_{\delta_R}}{\Delta_{Lat}} = \frac{A_\psi s'^3 + B_\psi s'^2 + C_\psi s' + D_\psi}{s'(As'^4 + Bs'^3 + Cs'^2 + Ds' + E)} \tag{76}$$

Figure 4-4 is a flow diagram of the computer program HORLIN.FORT (listed in Appendix C). The values of the coefficients and roots for the lateral transfer functions were calculated with HORLIN.FORT and are listed in Table 4-2. The coefficients and the roots of the roll polynomial are equal to zero because of the vehicle's symmetry in both the vertical and horizontal planes. The transfer functions for the sway and yaw motion are written (after cancellation):

$$v(s)/\delta r(s) = \frac{(S + 4.38)}{(S + 2.69)(S + .45)} \tag{77}$$

$$\psi(s)/\delta r(s) = \frac{(S + .81)}{(S + 2.69)(S + .45)} \tag{78}$$

v(t) and $\psi(t)$ were determined:

$$v(t) = 0.58 - 0.62\exp(-.45t) \tag{79}$$

$$\psi(t) = .014 - 0.01t - 0.012\exp(-.45t) \tag{80}$$

Plots of v(t) and $\psi(t)$ are shown in Figures 4.5 and 4.6. These plots show that the vehicle is stable and behaves in a predictable manner in the horizontal plane for small deflections of the rudder.

In order to maximize the use of the installed energy, the control surfaces should be redesigned to reduce their area, which would reduce the drag on the vehicle and thereby

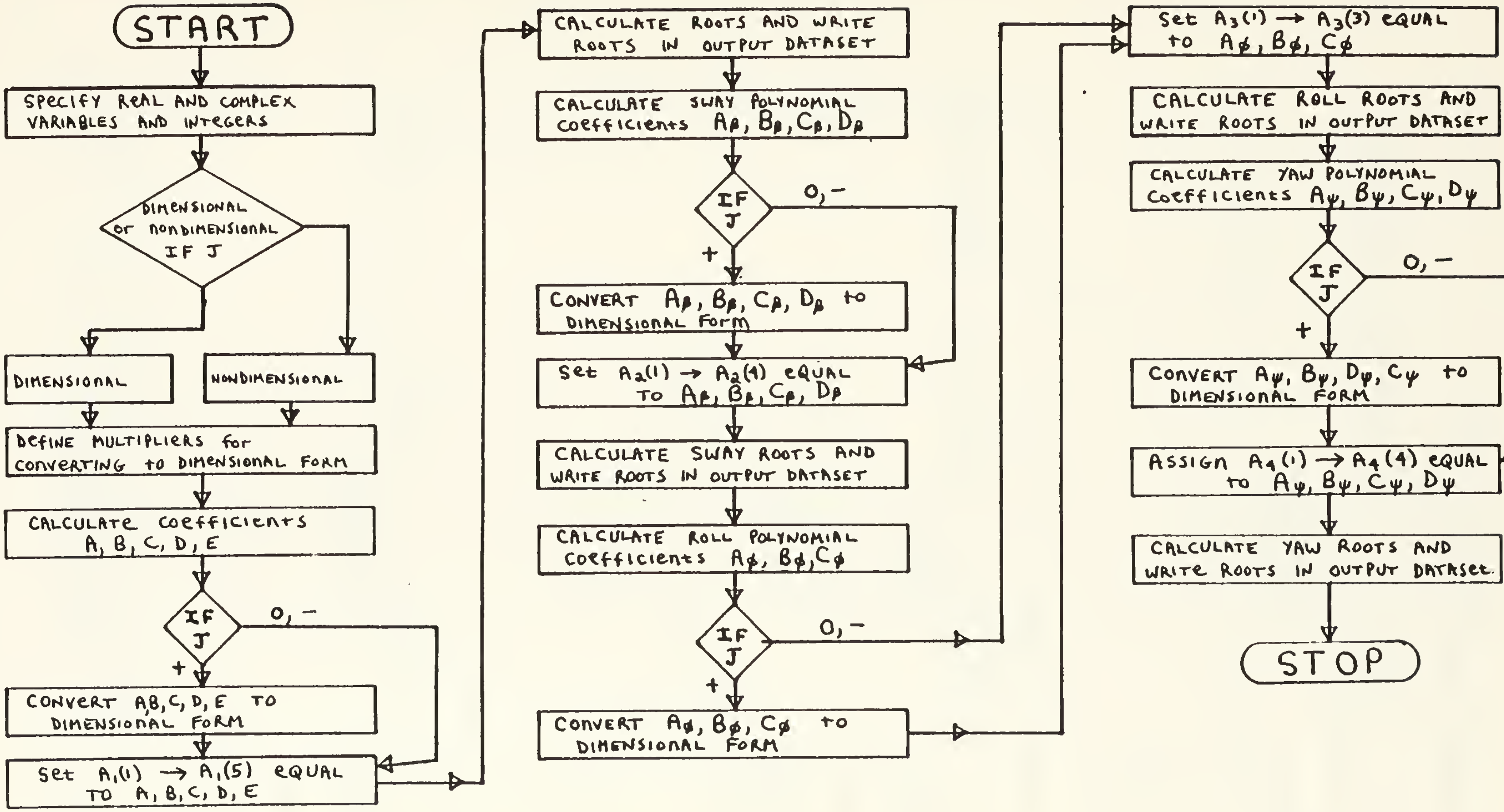


FIGURE 4.4 FLOWCHART FOR HORLIN.FORT

```
OUTPUT VALUES ARE IN DIMENSIONAL FORM
COEFFS. OF CHARACTERISTIC EQUATION

      A              B              C              D              E
  0.301377E-07  0.110544E-06  0.899080E-06  0.257019E-05  0.976523E-06

ROOTS OF CHARACTERISTIC EQUATION

         REAL            IMAGINARY
       -0.264699         5.186463

       -0.264699        -5.186463

       -2.692328         0.0

       -0.446244         0.0

SIDESLIP POLYNOMIAL COEFFS.

      ABETA          BBETA          CBETA          DBETA
  0.279417E-07  0.137099E-06  0.818321E-06  0.329856E-05
ROOTS
         REAL            IMAGINARY
       -4.377226         0.0

       -0.264695        -5.186463

       -0.264695         5.186463

ROLL POLYNOMIAL COEFFS.

      APHI           BPHI           CPHI
  0.0           0.0           0.0
ROOTS
         REAL            IMAGINARY
****************         0.0

****************         0.0

YAW POLYNOMIAL COEFFS.

      APSI           BPSI           CPSI           DPSI
 -0.269345E-07 -0.362082E-07 -0.738029E-06 -0.591960E-06
ROOTS
         REAL            IMAGINARY
       -0.264697         5.186465

       -0.264697        -5.186465

       -0.814914         0.0
```

TABLE 4-2 ROOTS OF HORIZONTAL TRANSFER FUNCTIONS

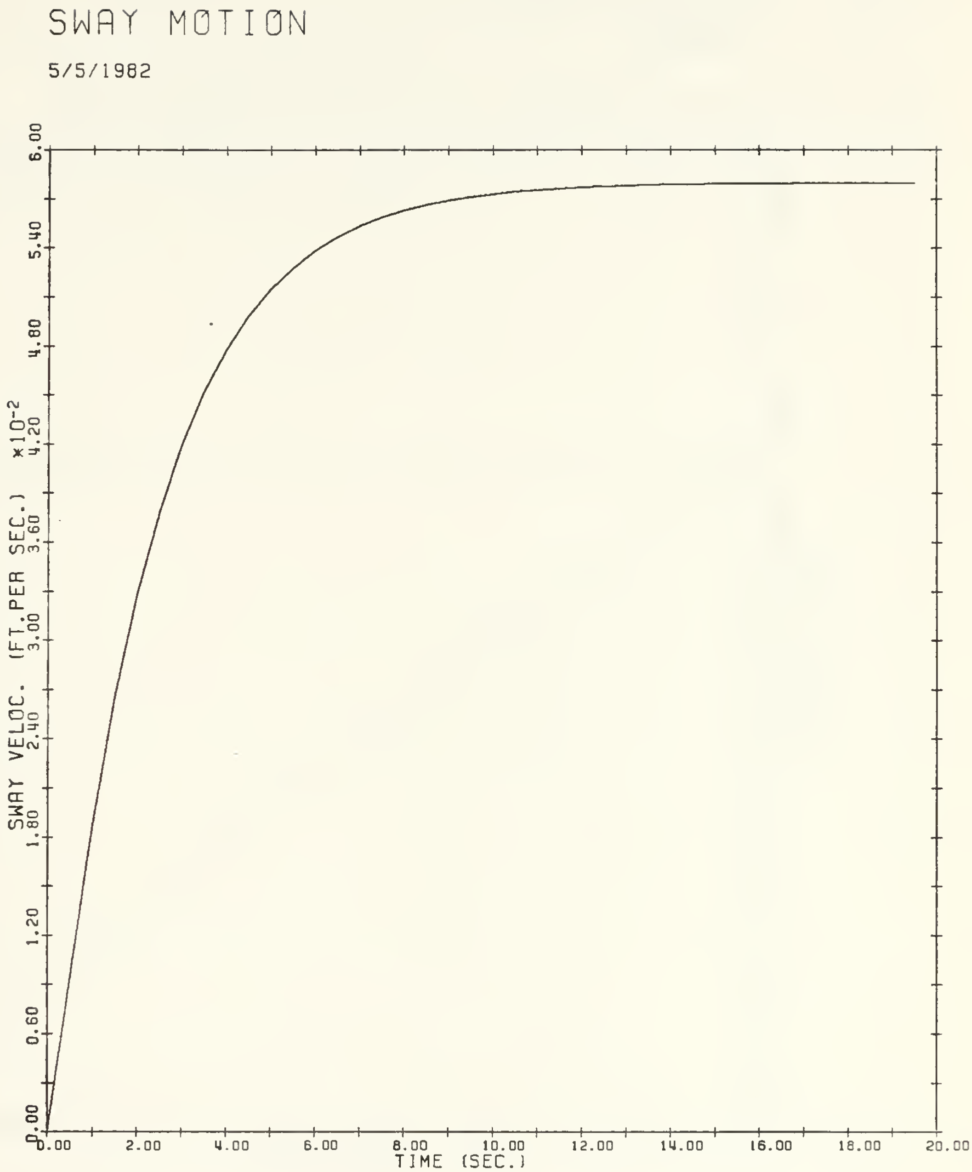


FIGURE 4.5 SWAY MOTION FOR A ONE DEGREE RUDDER DEFLECTION

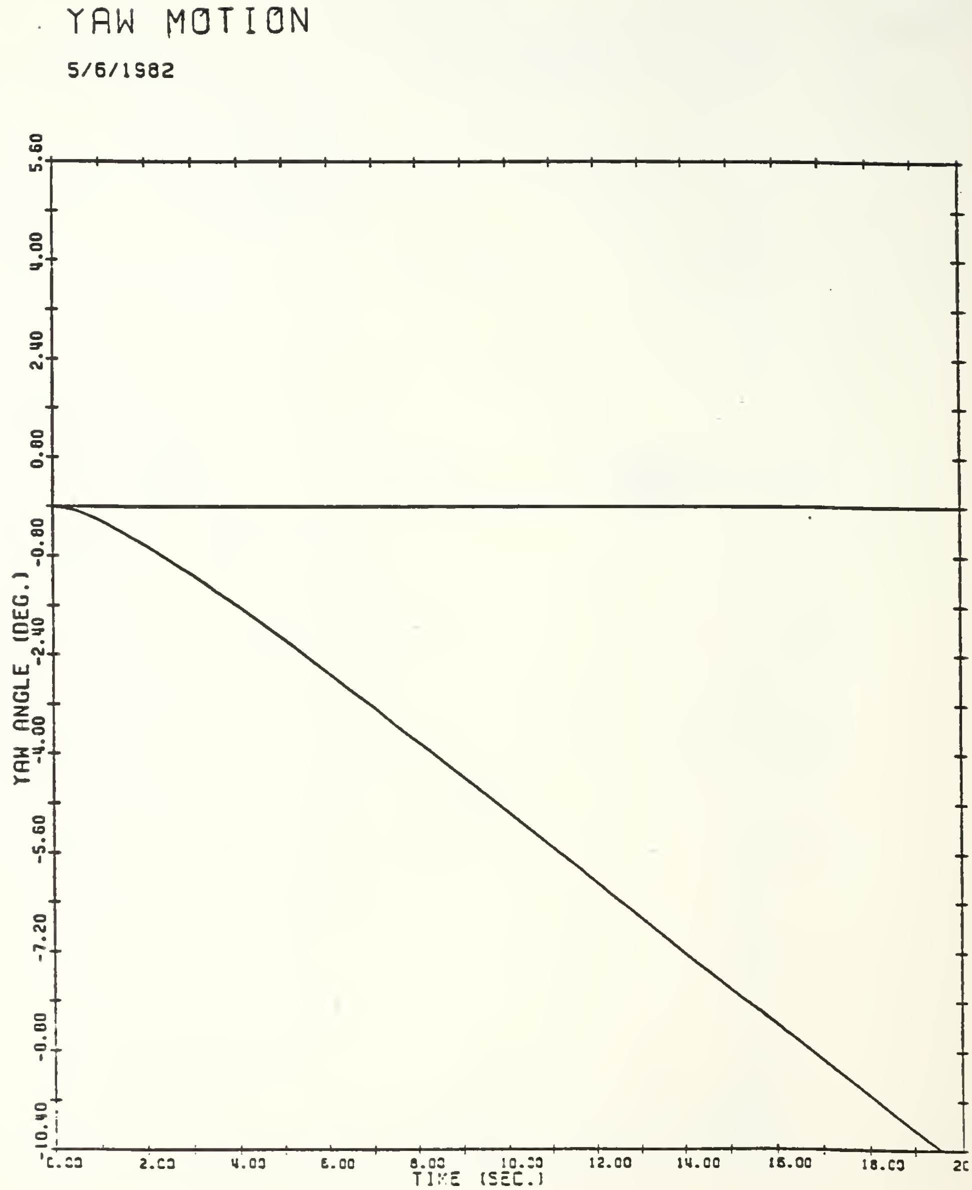


FIGURE 4.6 YAW MOTION FOR A ONE DEGREE RUDDER DEFLECTION

maximize mission range or time. After selecting a new fin size and shape, the affected calculations in Chapter III should be repeated to determine new hydrodynamic coefficients. Then the analysis of this chapter should be repeated. This sequence should be repeated until the size of the control surfaces is the smallest which will provide the desired control characteristics.

Chapter V

APPLICATION OF MODERN CONTROL THEORY

After analyzing the vehicle using classical control theory, the next step was to apply the principals of modern control theory which uses state space, state variables and state vectors to describe the dynamic system which is to be controlled. Modern control theory can be used to design control systems which have multiple inputs and multiple outputs.

The state variables are a set of variables of interest, X<i>, which completely describe the state of the system at any fixed time. The finite number, n, of state variables which can completely describe the system at any instant form an n component state vector:

$$\vec{X} = (X_1, X_2, ..X\langle n\rangle)$$

The inputs to the system are represented by a control vector, $\vec{U}$.

The state equations which represent the system are then written in the form:

$$\dot{\vec{X}} = \bar{A}\,\vec{X} + \bar{B}\,\vec{U}$$

$$\dot{\vec{Y}} = \bar{C}\,\vec{X} + \bar{D}\,\vec{U}$$

where A,B,C, and D are coefficient matrices and Y(t) is the output vector. The state variables are not a unique set. That is, various sets of state variables can be used. Where possible, it is advantageous to choose as the state variables those variables which have physical significance and which can be measured.

The next step in the design sequence was to model the nonlinear motion of the vehicle in six degrees of freedom. Appendix D contains a listing of the six nonlinear equations which were proposed as the standard equations of motion for submarine simulation[9].

Appendix E[10] contains the listing of a computer subroutine which uses these equations to calculate the values of the vehicle's linear and angular velocities in both the body coordinate system and the earth coordinate system. The flow chart for this subroutine is shown in Figure 5.1.

This completes Phases I through V of the control system design process as discussed in Chapter I. Additional work on this topic should begin with Phase VI and procede to the final control system design.

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9 Gerter, M. and Hagen, G.R.; <u>Standard Equations of Motion for Submarine Simulation</u>; NSRDC Report No. 2510; Washington D.C. ; June, 1967

10 Lee, Jang Gyu; The Charles Stark Draper Laboratory, Inc.; February, 1982

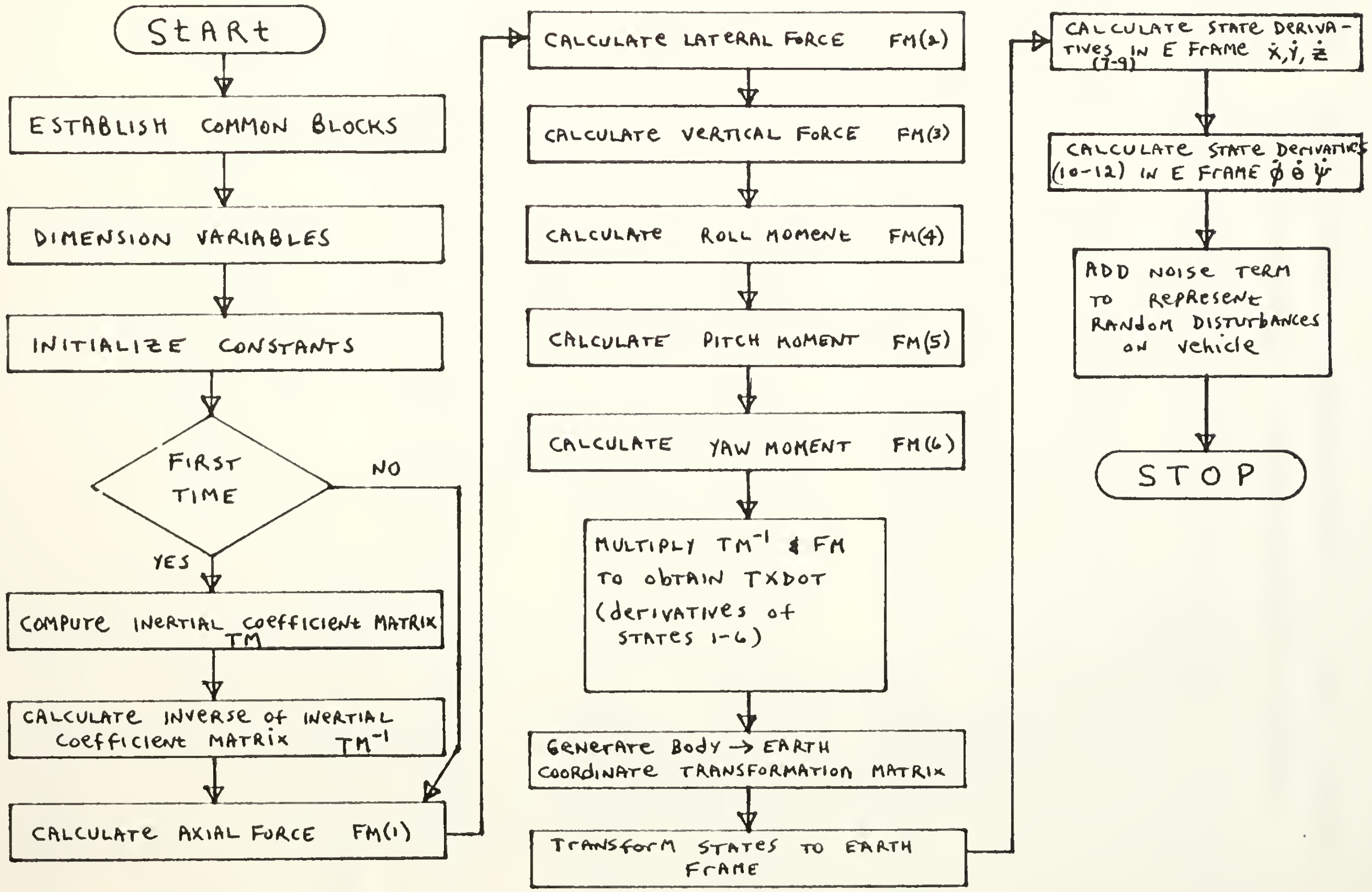

FIGURE 5.1 FLOWCHART FOR SUBROUTINE TRUMOD

Chapter VI

CONCLUSIONS

A general procedure for the control system design of a small unmanned, untethered, underwater vehicle was formulated. The early phases of the procedure were accomplished for the vehicle. accomplished for a vehicle.

The linearized equations of motion in six degrees of freedom for small perturbations about an equilibrium condition of straight ahead motion were formulated, and values for each of the hydrodynamic coefficients in these equations were calculated. The vehicle's response to a small control surface deflection was analyzed by forming the transfer functions for each of the equations and then solving for the vehicle response in the time domain. The initial design of the control surfaces for the selected vehicle size and shape was conservative. Smaller control surfaces should be designed and the analysis should be repeated until the smallest control surfaces which provide satisfactory vehicle control are achieved.

A model of the vehicle was formulate using the nonlinear equations of motion. Additional work in the area of this thesis would be to complete a nonlinear model of the vehicle and its control system. Modeling of the vehicle's sensors

and measurement noise terms and constructing an observer to calculate non-observable states are required to combine with the nonlinear vehicle model and input terms.

This complete nonlinear model should then be used to predict the vehicle's response in real time. An actual control system should then be designed that has the desired characteristics to control the vehicle within the specified tolerances.

APPENDIX A

NOTATION

Symbol	Dimensionless Form	Definition
B	$B' = \frac{B}{\frac{1}{2}\rho \ell^2 U^2}$	Buoyancy force, positive upward
CB		Center of buoyancy of submarine
CG		Center of mass of submarine
I_x	$I_x' = \frac{I_x}{\frac{1}{2}\rho \ell^5}$	Moment of inertia of submarine about x axis
I_y	$I_y' = \frac{I_y}{\frac{1}{2}\rho \ell^5}$	Moment of inertia of submarine about y axis
I_z	$I_z' = \frac{I_z}{\frac{1}{2}\rho \ell^5}$	Moment of inertia of submarine about z axis
I_{xy}	$I_{xy}' = \frac{I_{xy}}{\frac{1}{2}\rho \ell^5}$	Product of inertia about xy axis
I_{yz}	$I_{yz}' = \frac{I_{yz}}{\frac{1}{2}\rho \ell^5}$	Product of inertia about yz axes
I_{zx}	$I_{zx}' = \frac{I_{zx}}{\frac{1}{2}\rho \ell^5}$	Product of inertia about zx axes
K	$K' = \frac{K}{\frac{1}{2}\rho \ell^3 U^2}$	Hydrodynamic moment component about x axis (rolling moment)
K_*	$K_*' = \frac{K_*}{\frac{1}{2}\rho \ell^3 U^2}$	Rolling moment when body angle (α, β) and control surface angles are zero
$K_{*\eta}$	$K_{*\eta}' = \frac{K_{*\eta}}{\frac{1}{2}\rho \ell^3 U^2}$	Coefficient used in representing K_* as a function of $(\eta-1)$
K_p	$K_p' = \frac{K_p}{\frac{1}{2}\rho \ell^4 U}$	First order coefficient used in representing K as a function of p
$K_{\dot{p}}$	$K_{\dot{p}}' = \frac{K_{\dot{p}}}{\frac{1}{2}\rho \ell^5}$	Coefficient used in representing K as a function of $\dot{p}$
$K_{p\|p\|}$	$K_{p\|p\|}' = \frac{K_{p\|p\|}}{\frac{1}{2}\rho \ell^5}$	Second order coefficient used in representing K as a function of p
K_{pq}	$K_{pq}' = \frac{K_{pq}}{\frac{1}{2}\rho \ell^5}$	Coefficient used in representing K as a function of the product pq

K_{qr}	$K_{qr}' = \dfrac{K_{qr}}{\frac{1}{2}\rho \ell^5}$	Coefficient used in representing K as a function of the product qr
K_r	$K_r' = \dfrac{K_r}{\frac{1}{2}\rho \ell^4 U}$	First order coefficient used in representing K as a function of r
$K_{\dot{r}}$	$K_{\dot{r}}' = \dfrac{K_{\dot{r}}}{\frac{1}{2}\rho \ell^5}$	Coefficient used in representing K as a function of $\dot{r}$
K_v	$K_v' = \dfrac{K_v}{\frac{1}{2}\rho \ell^3 U}$	First order coefficient used in representing K as a function of v
$K_{\dot{v}}$	$K_{\dot{v}}' = \dfrac{K_{\dot{v}}}{\frac{1}{2}\rho \ell^4}$	Coefficient used in representing K as a function of $\dot{v}$
$K_{v\|v\|}$	$K_{v\|v\|}' = \dfrac{K_{v\|v\|}}{\frac{1}{2}\rho \ell^3}$	Second order coefficient used in representing K as a function of v
K_{vq}	$K_{vq}' = \dfrac{K_{vq}}{\frac{1}{2}\rho \ell^4}$	Coefficient used in representing K as a function of the product vq
K_{vw}	$K_{vw}' = \dfrac{K_{vw}}{\frac{1}{2}\rho \ell^3}$	Coefficient used in representing K as a function of the product vw
K_{wp}	$K_{wp}' = \dfrac{K_{wp}}{\frac{1}{2}\rho \ell^4}$	Coefficient used in representing K as a function of the product wp
K_{wr}	$K_{wr}' = \dfrac{K_{wr}}{\frac{1}{2}\rho \ell^4}$	Coefficient used in representing K as a function of the product wr
$K_{\delta r}$	$K_{\delta r}' = \dfrac{K_{\delta r}}{\frac{1}{2}\rho \ell^3 U^2}$	First order coefficient used in representing K as a function of δ_r
ℓ	$\ell' = 1$	Overall length of submarine
m	$m' = \dfrac{m}{\frac{1}{2}\rho \ell^3}$	Mass of submarine, including water in free-flooding spaces
M	$M' = \dfrac{M}{\frac{1}{2}\rho \ell^3 U^2}$	Hydrodynamic moment component about y axis (pitching moment)
M_*	$M_*' = \dfrac{M_*}{\frac{1}{2}\rho \ell^3 U^2}$	Pitching moment when body angles (α, β) and control surface angles are zero
M_{pp}	$M_{pp}' = \dfrac{M_{pp}}{\frac{1}{2}\rho \ell^5}$	Second order coefficient used in representing M as a function of p. First order coefficient is zero.
M_q	$M_q' = \dfrac{M_q}{\frac{1}{2}\rho \ell^4 U}$	First order coefficient used in representing M as a function of q
$M_{q\eta}$	$M_{q\eta}' = \dfrac{M_{q\eta}}{\frac{1}{2}\rho \ell^4 U}$	First order coefficient used in representing M_q as a function of $(\eta-1)$
$M_{\dot{q}}$	$M_{\dot{q}}' = \dfrac{M_{\dot{q}}}{\frac{1}{2}\rho \ell^5}$	Coefficient used in representing M as a function of $\dot{q}$

$M_{q\|q\|}$	$M_{q\|q\|}' = \frac{M_{q\|q\|}}{\frac{1}{2}\rho \ell^5}$	Second order coefficient used in representing M as a function of q
$M_{\|q\|\delta s}$	$M_{\|q\|\delta s}' = \frac{M_{\|q\|\delta s}}{\frac{1}{2}\rho \ell^4 U}$	Coefficient used in representing $M_{\delta s}$ as a function q
M_{rp}	$M_{rp}' = \frac{M_{rp}}{\frac{1}{2}\rho \ell^5}$	Coefficient used in representing M as a function of the product rp
M_{rr}	$M_{rr}' = \frac{M_{rr}}{\frac{1}{2}\rho \ell^5}$	Second order coefficient used in representing M as a function of r. First order coefficient is zero
M_{vp}	$M_{vp}' = \frac{M_{vp}}{\frac{1}{2}\rho \ell^4}$	Coefficient used in representing M as a function of the product vp
M_{vr}	$M_{vr}' = \frac{M_{vr}}{\frac{1}{2}\rho \ell^4}$	Coefficient used in representing M as a function of the product vr
M_{vv}	$M_{vv}' = \frac{M_{vv}}{\frac{1}{2}\rho \ell^3}$	Second order coefficient used in representing M as a function of v
M_w	$M_w' = \frac{M_w}{\frac{1}{2}\rho \ell^3 U}$	First order coefficient used in representing M as a function of w
$M_{w\eta}$	$M_{w\eta}' = \frac{M_{w\eta}}{\frac{1}{2}\rho \ell^3 U}$	First order coefficient used in representing M_w as a function of $(\eta - 1)$
$M_{\dot{w}}$	$M_{\dot{w}}' = \frac{M_{\dot{w}}}{\frac{1}{2}\rho \ell^4}$	Coefficient used in representing M as a function of $\dot{w}$
$M_{\|w\|}$	$M_{\|w\|}' = \frac{M_{\|w\|}}{\frac{1}{2}\rho \ell^3 U}$	First order coefficient used in representing M as a function of w; equal to zero for symmetrical function
$M_{\|w\|q}$	$M_{\|w\|q}' = \frac{M_{\|w\|q}}{\frac{1}{2}\rho \ell^4}$	Coefficient used in representing M_q as a function of w
$M_{w\|w\|}$	$M_{w\|w\|}' = \frac{M_{w\|w\|}}{\frac{1}{2}\rho \ell^3}$	Second order coefficient used in representing M as a function of w
$M_{w\|w\|\eta}$	$M_{w\|w\|\eta}' = \frac{M_{w\|w\|\eta}}{\frac{1}{2}\rho \ell^3}$	First order coefficient used in representing $M_{w\|w\|}$ as a function of $(\eta - 1)$
M_{ww}	$M_{ww}' = \frac{M_{ww}}{\frac{1}{2}\rho \ell^3}$	Second order coefficient used in representing M as a function of w; equal to zero for symmetrical function
$M_{\delta b}$	$M_{\delta b}' = \frac{M_{\delta b}}{\frac{1}{2}\rho \ell^3 U^2}$	First order coefficient used in representing M as a function of δ_b
$M_{\delta s}$	$M_{\delta s}' = \frac{M_{\delta s}}{\frac{1}{2}\rho \ell^3 U^2}$	First order coefficient used in representing M as a function of δ_s
$M_{\delta s\eta}$	$M_{\delta s\eta}' = \frac{M_{\delta s\eta}}{\frac{1}{2}\rho \ell^3 U^2}$	First order coefficient used in representing $M_{\delta s}$ as a function of $(\eta - 1)$

N	$N' = \frac{N}{\frac{1}{2}\rho \ell^3 U^2}$	Hydrodynamic moment component about z axis (yawing moment)
N_*	$N_*' = \frac{N_*}{\frac{1}{2}\rho \ell^3 U^2}$	Yawing moment when body angles (α, β) and control surface angles are zero
N_p	$N_p' = \frac{N_p}{\frac{1}{2}\rho \ell^4 U}$	First order coefficient used in representing N as a function of p
$N_{\dot{p}}$	$N_{\dot{p}}' = \frac{N_{\dot{p}}}{\frac{1}{2}\rho \ell^5}$	Coefficient used in representing N as a function of $\dot{p}$
N_{pq}	$N_{pq}' = \frac{N_{pq}}{\frac{1}{2}\rho \ell^5}$	Coefficient used in representing N as a function of the product pq
N_{qr}	$N_{qr}' = \frac{N_{qr}}{\frac{1}{2}\rho \ell^5}$	Coefficient used in representing N as a function of the product qr
N_r	$N_r' = \frac{N_r}{\frac{1}{2}\rho \ell^4 U}$	First order coefficient used in representing N as a function of r
$N_{r\eta}$	$N_{r\eta}' = \frac{N_{r\eta}}{\frac{1}{2}\rho \ell^4 U}$	First order coefficient used in representing N_r as a function of (η-1)
$N_{\dot{r}}$	$N_{\dot{r}}' = \frac{N_{\dot{r}}}{\frac{1}{2}\rho \ell^5}$	Coefficient used in representing N as a function of $\dot{r}$
$N_{r\|r\|}$	$N_{r\|r\|}' = \frac{N_{r\|r\|}}{\frac{1}{2}\rho \ell^5}$	Second order coefficient used in representing N as a function of r
$N_{\|r\|\delta r}$	$N_{\|r\|\delta r}' = \frac{N_{\|r\|\delta r}}{\frac{1}{2}\rho \ell^4 U}$	Coefficient used in representing $N_{\delta r}$ as a function of r
N_v	$N_v' = \frac{N_v}{\frac{1}{2}\rho \ell^3 U}$	First order coefficient used in representing N as a function of v
$N_{v\eta}$	$N_{v\eta}' = \frac{N_{v\eta}}{\frac{1}{2}\rho \ell^3 U}$	First order coefficient used in representing N_v as a function of (η-1)
$N_{\dot{v}}$	$N_{\dot{v}}' = \frac{N_{\dot{v}}}{\frac{1}{2}\rho \ell^4}$	Coefficient used in representing N as a function of $\dot{v}$
N_{vq}	$N_{vq}' = \frac{N_{vq}}{\frac{1}{2}\rho \ell^4}$	Coefficient used in representing N as a function of the product vq
$N_{\|v\|r}$	$N_{\|v\|r}' = \frac{N_{\|v\|r}}{\frac{1}{2}\rho \ell^4}$	Coefficient used in representing N_r as a function of v
$N_{v\|v\|}$	$N_{v\|v\|}' = \frac{N_{v\|v\|}}{\frac{1}{2}\rho \ell^3}$	Second order coefficient used in representing N as a function of v
$N_{v\|v\|\eta}$	$N_{v\|v\|\eta}' = \frac{N_{v\|v\|\eta}}{\frac{1}{2}\rho \ell^3}$	First order coefficient used in representing $N_{v\|v\|}$ as a function of (η-1)

N_{vw}	$N_{vw}' = \frac{N_{vw}}{\frac{1}{2}\rho \ell^3}$	Coefficient used in representing N as a function of the product vw
N_{wp}	$N_{wp}' = \frac{N_{wp}}{\frac{1}{2}\rho \ell^4}$	Coefficient used in representing N as a function of the product wp
N_{wr}	$N_{wr}' = \frac{N_{wr}}{\frac{1}{2}\rho \ell^4}$	Coefficient used in representing N as a function of the product wr
$N_{\delta r}$	$N_{\delta r}' = \frac{N_{\delta r}}{\frac{1}{2}\rho \ell^3 U^2}$	First order coefficient used in representing N as a function of δ_r
$N_{\delta r\eta}$	$N_{\delta r\eta}' = \frac{N_{\delta r\eta}}{\frac{1}{2}\rho \ell^3 U^2}$	First order coefficient used in representing $N_{\delta r}$ as a function of $(\eta - 1)$
p	$p' = \frac{p\ell}{U}$	Angular velocity component about x axis relative to fluid (roll)
$\dot{p}$	$\dot{p}' = \frac{\dot{p}\ell^2}{U^2}$	Angular acceleration component about x axis relative to fluid
q	$q' = \frac{q\ell}{U}$	Angular velocity component about y axis relative to fluid (pitch)
$\dot{q}$	$\dot{q}' = \frac{\dot{q}\ell^2}{U^2}$	Angular acceleration component about y axis relative to fluid
r	$r' = \frac{r\ell}{U}$	Angular velocity component about z axis relative to fluid (yaw)
$\dot{r}$	$\dot{r}' = \frac{\dot{r}\ell^2}{U^2}$	Angular acceleration component about z axis relative to fluid
U	$U' = \frac{U}{U}$	Linear velocity of origin of body axes relative to fluid
u	$u' = \frac{u}{U}$	Component of U in direction of the x axis
$\dot{u}$	$\dot{u}' = \frac{\dot{u}\ell}{U^2}$	Time rate of change of u in direction of the x axis
u_c	$u_c' = \frac{u_c}{U}$	Command speed: steady value of ahead speed component u for a given propeller rpm when body angles (α, β) and control surface angles are zero. Sign changes with propeller reversal
v	$v' = \frac{v}{U}$	Component of U in direction of the y axis
$\dot{v}$	$\dot{v}' = \frac{\dot{v}\ell}{U^2}$	Time rate of change of v in direction of the y axis

w	$w' = \frac{w}{U}$	Component of U in direction of the z axis
$\dot{w}$	$\dot{w}' = \frac{\dot{w}\ell}{U^2}$	Time rate of change of w in direction of the z axis
W	$W' = \frac{W}{\frac{1}{2}\rho\ell^2U^2}$	Weight, including water in free flooding spaces
x	$x' = \frac{x}{\ell}$	Longitudinal body axis; also the coordinate of a point relative to the origin of body axes
x_B	$x_B' = \frac{x_B}{\ell}$	The x coordinate of CB
x_G	$x_G' = \frac{x_G}{\ell}$	The x coordinate of CG
x_o	$x_o' = \frac{x_o}{\ell}$	A coordinate of the displacement of CG relative to the origin of a set of fixed axes
X	$X' = \frac{X}{\frac{1}{2}\rho\ell^2U^2}$	Hydrodynamic force component along x axis (longitudinal, or axial, force)
X_{qq}	$X_{qq}' = \frac{X_{qq}}{\frac{1}{2}\rho\ell^4}$	Second order coefficient used in representing X as a function of q. First order coefficient is zero
X_{rp}	$X_{rp}' = \frac{X_{rp}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing X as a function of the product rp
X_{rr}	$X_{rr}' = \frac{X_{rr}}{\frac{1}{2}\rho\ell^4}$	Second order coefficient used in representing X as a function of r. First order coefficient is zero
$X_{\dot{u}}$	$X_{\dot{u}}' = \frac{X_{\dot{u}}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing X as a function of $\dot{u}$
X_{uu}	$X_{uu}' = \frac{X_{uu}}{\frac{1}{2}\rho\ell^2}$	Second order coefficient used in representing X as a function of u in the non-propelled case. First order coefficient is zero
X_{vr}	$X_{vr}' = \frac{X_{vr}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing X as a function of the product vr
X_{vv}	$X_{vv}' = \frac{X_{vv}}{\frac{1}{2}\rho\ell^2}$	Second order coefficient used in representing X as a function of v, First order coefficient is zero
$X_{vv\eta}$	$X_{vv\eta}' = \frac{X_{vv\eta}}{\frac{1}{2}\rho\ell^2}$	First order coefficient used in representing X_{vv} as a function of $(\eta - 1)$
X_{wq}	$X_{wq}' = \frac{X_{wq}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing X as a function of the product wq

X_{ww}	$X_{ww}' = \frac{X_{ww}}{\frac{1}{2}\rho \ell^2}$	Second order coefficient used in representing X as a function of w. First order coefficient is zero
$X_{ww\eta}$	$X_{ww\eta}' = \frac{X_{ww\eta}}{\frac{1}{2}\rho \ell^2}$	First order coefficient used in representing X_{ww} as a function of $(\eta - 1)$
$X_{\delta b \delta b}$	$X_{\delta b \delta b}' = \frac{X_{\delta b \delta b}}{\frac{1}{2}\rho \ell^2 U^2}$	Second order coefficient used in representing X as a function of δ_b. First order coefficient is zero
$X_{\delta r \delta r}$	$X_{\delta r \delta r}' = \frac{X_{\delta r \delta r}}{\frac{1}{2}\rho \ell^2 U^2}$	Second order coefficient used in representing X as a function of δ_r. First order coefficient is zero
$X_{\delta r \delta r \eta}$	$X_{\delta r \delta r \eta}' = \frac{X_{\delta r \delta r \eta}}{\frac{1}{2}\rho \ell^2 U^2}$	First order coefficient used in representing $X_{\delta r \delta r}$ as a function of $(\eta - 1)$
$X_{\delta s \delta s}$	$X_{\delta s \delta s}' = \frac{X_{\delta s \delta s}}{\frac{1}{2}\rho \ell^2 U^2}$	Second order coefficient used in representing X as a function of δ_s. First order coefficient is zero
$X_{\delta s \delta s \eta}$	$X_{\delta s \delta s \eta}' = \frac{X_{\delta s \delta s \eta}}{\frac{1}{2}\rho \ell^2 U^2}$	First order coefficient used in representing $X_{\delta s \delta s}$ as a function of $(\eta - 1)$
y	$y' = \frac{y}{\ell}$	Lateral body axis; also the coordinate of a point relative to the origin of body axes
y_B	$y_B' = \frac{y_B}{\ell}$	The y coordinate of CB
y_G	$y_G' = \frac{y_G}{\ell}$	The y coordinate of CG
y_o	$y_o' = \frac{y_o}{\ell}$	A coordinate of the displacement of CG relative to the origin of a set of fixed axes
Y	$Y' = \frac{Y}{\frac{1}{2}\rho \ell^2 U^2}$	Hydrodynamic force component along y axis (lateral force)
Y_*	$Y_*' = \frac{Y}{\frac{1}{2}\rho \ell^2 U^2}$	Lateral force when body angles (α, β) and control surface angles are zero
Y_p	$Y_p' = \frac{Y_p}{\frac{1}{2}\rho \ell^3 U}$	First order coefficient used in representing Y as a function of p
$Y_{\dot{p}}$	$Y_{\dot{p}}' = \frac{Y_{\dot{p}}}{\frac{1}{2}\rho \ell^4}$	Coefficient used in representing Y as a function of $\dot{p}$
$Y_{p\|p\|}$	$Y_{p\|p\|}' = \frac{Y_{p\|p\|}}{\frac{1}{2}\rho \ell^4}$	Second order coefficient used in representing Y as a function of p

Y_{pq}	$Y_{pq}' = \dfrac{Y_{pq}}{\frac{1}{2}\rho \ell^4}$	Coefficient used in representing Y as a function of the product pq
Y_{qr}	$Y_{qr}' = \dfrac{Y_{qr}}{\frac{1}{2}\rho \ell^4}$	Coefficient used in representing Y as a function of the product qr
Y_r	$Y_r' = \dfrac{Y_r}{\frac{1}{2}\rho \ell^3 U}$	First order coefficient used in representing Y as a function of r
$Y_{r\eta}$	$Y_{r\eta}' = \dfrac{Y_{r\eta}}{\frac{1}{2}\rho \ell^3 U}$	First order coefficient used in representing Y_r as a function of $(\eta - 1)$
$Y_{\dot{r}}$	$Y_{\dot{r}}' = \dfrac{Y_{\dot{r}}}{\frac{1}{2}\rho \ell^4}$	Coefficient used in representing Y as a function of $\dot{r}$
$Y_{\lvert r \rvert \delta r}$	$Y_{\lvert r \rvert \delta r}' = \dfrac{Y_{\lvert r \rvert \delta r}}{\frac{1}{2}\rho \ell^3 U}$	Coefficient used in representing $Y_{\delta r}$ as a function of r
Y_v	$Y_v' = \dfrac{Y_v}{\frac{1}{2}\rho \ell^2 U}$	First order coefficient used in representing Y as a function of v
$Y_{v\eta}$	$Y_{v\eta}' = \dfrac{Y_{v\eta}}{\frac{1}{2}\rho \ell^2 U}$	First order coefficient used in representing Y_v as a function of $(\eta - 1)$
$Y_{\dot{v}}$	$Y_{\dot{v}}' = \dfrac{Y_{\dot{v}}}{\frac{1}{2}\rho \ell^3}$	Coefficient used in representing Y as a function of $\dot{v}$
Y_{vq}	$Y_{vq}' = \dfrac{Y_{vq}}{\frac{1}{2}\rho \ell^3}$	Coefficient used in representing Y as a function of the product vq
$Y_{v \lvert r \rvert}$	$Y_{v \lvert r \rvert}' = \dfrac{Y_{v \lvert r \rvert}}{\frac{1}{2}\rho \ell^3}$	Coefficient used in representing Y_v as a function of r
$Y_{v \lvert v \rvert}$	$Y_{v \lvert v \rvert}' = \dfrac{Y_{v \lvert v \rvert}}{\frac{1}{2}\rho \ell^2}$	Second order coefficient used in representing Y as a function of v
$Y_{v \lvert v \rvert \eta}$	$Y_{v \lvert v \rvert \eta}' = \dfrac{Y_{v \lvert v \rvert \eta}}{\frac{1}{2}\rho \ell^2}$	First order coefficient used in representing $Y_{v \lvert v \rvert}$ as a function of $(\eta - 1)$
Y_{vw}	$Y_{vw}' = \dfrac{Y_{vw}}{\frac{1}{2}\rho \ell^2}$	Coefficient used in representing Y as a function of the product vw
Y_{wp}	$Y_{wp}' = \dfrac{Y_{wp}}{\frac{1}{2}\rho \ell^3}$	Coefficient used in representing Y as a function of the product wp
Y_{wr}	$Y_{wr}' = \dfrac{Y_{wr}}{\frac{1}{2}\rho \ell^3}$	Coefficient used in representing Y as a function of the product wr
$Y_{\delta r}$	$Y_{\delta r}' = \dfrac{Y_{\delta r}}{\frac{1}{2}\rho \ell^2 U^2}$	First order coefficient used in representing Y as a function of δr
$Y_{\delta r\eta}$	$Y_{\delta r\eta}' = \dfrac{Y_{\delta r\eta}}{\frac{1}{2}\rho \ell^2 U^2}$	First order coefficient used in representing $Y_{\delta r}$ as a function of $(\eta - 1)$

z	$z' = \frac{z}{\ell}$	Normal body axis; also the coordinate of a point relative to the origin of body axes
z_B	$z_B' = \frac{z_B}{\ell}$	The z coordinate of CB
z_G	$z_G' = \frac{z_G}{\ell}$	The z coordinate of CG
z_o	$z_o' = \frac{z_o}{\ell}$	A coordinate of the displacement of CG relative to the origin of a set of fixed axes
Z	$Z' = \frac{Z}{\frac{1}{2}\rho\ell^2U^2}$	Hydrodynamic force component along z axis (normal force)
Z_*	$Z_*' = \frac{Z_*}{\frac{1}{2}\rho\ell^2U^2}$	Normal force when body angles (α, β) and control surface angles are zero
Z_{pp}	$Z_{pp}' = \frac{Z_{pp}}{\frac{1}{2}\rho\ell^4}$	Second order coefficient used in representing Z as a function of p. First order coefficient is zero
Z_q	$Z_q' = \frac{Z_q}{\frac{1}{2}\rho\ell^3U}$	First order coefficient used in representing Z as a function of q
$Z_{q\eta}$	$Z_{q\eta}' = \frac{Z_{q\eta}}{\frac{1}{2}\rho\ell^3U}$	First order coefficient used in representing Z_q as a function of $(\eta-1)$
$Z_{\dot{q}}$	$Z_{\dot{q}}' = \frac{Z_{\dot{q}}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing Z as a function of $\dot{q}$
$Z_{\lvert q\rvert\delta s}$	$Z_{\lvert q\rvert\delta s}' = \frac{Z_{\lvert q\rvert\delta s}}{\frac{1}{2}\rho\ell^3U}$	Coefficient used in representing $Z_{\delta s}$ as a function of q
Z_{rp}	$Z_{rp}' = \frac{Z_{rp}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing Z as a function of the product rp
Z_{rr}	$Z_{rr}' = \frac{Z_{rr}}{\frac{1}{2}\rho\ell^4}$	Second order coefficient used in representing Z as a function of r. First order coefficient is zero
Z_w	$Z_w' = \frac{Z_w}{\frac{1}{2}\rho\ell^2U}$	First order coefficient used in representing Z as a function of w
$Z_{w\eta}$	$Z_{w\eta}' = \frac{Z_{w\eta}}{\frac{1}{2}\rho\ell^2U}$	First order coefficient used in representing Z_w as a function of $(\eta-1)$
$Z_{\dot{w}}$	$Z_{\dot{w}}' = \frac{Z_{\dot{w}}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing Z as a function of $\dot{w}$
$Z_{\lvert w\rvert}$	$Z_{\lvert w\rvert}' = \frac{Z_{\lvert w\rvert}}{\frac{1}{2}\rho\ell^2U}$	First order coefficient used in representing Z as a function of w; equal to zero for symmetrical function
$Z_{w\lvert q\rvert}$	$Z_{w\lvert q\rvert}' = \frac{Z_{w\lvert q\rvert}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing Z_w as a function of q

$Z_{w\|w\|}$	$Z_{w\|w\|}' = \frac{Z_{w\|w\|}}{\frac{1}{2}\rho \ell^2}$	Second order coefficient used in representing Z as a function of w
$Z_{w\|w\|\eta}$	$Z_{w\|w\|\eta}' = \frac{Z_{w\|w\|\eta}}{\frac{1}{2}\rho \ell^2}$	First order coefficient used in representing $Z_{w\|w\|}$ as a function of $(\eta - 1)$
Z_{ww}	$Z_{ww}' = \frac{Z_{ww}}{\frac{1}{2}\rho \ell^2}$	Second order coefficient used in representing Z as a function of w; equal to zero for symmetrical function
$Z_{\delta b}$	$Z_{\delta b}' = \frac{Z_{\delta b}}{\frac{1}{2}\rho \ell^2 U^2}$	First order coefficient used in representing Z as a function of δ_b
$Z_{\delta s}$	$Z_{\delta s}' = \frac{Z_{\delta s}}{\frac{1}{2}\rho \ell^2 U^2}$	First order coefficient used in representing Z as a function of δ_s
$Z_{\delta s \eta}$	$Z_{\delta s \eta}' = \frac{Z_{\delta s \eta}}{\frac{1}{2}\rho \ell^2 U^2}$	First order coefficient used in representing $Z_{\delta s}$ as a function of $(\eta - 1)$
α		Angle of attack
β		Angle of drift
δ_b		Deflection of bowplane or sailplane
δ_r		Deflection of rudder
δ_s		Deflection of sternplane
η		The ratio $\frac{u_c}{U}$
θ		Angle of pitch
ψ		Angle of yaw
ϕ		Angle of roll
a_i, b_i, c_i		Sets of constants used in the representation of propeller thrust in the axial equation

APPENDIX B

EXPRESSIONS FOR THE LONGITUDINAL TRANSFER FUNCTION COEFFICIENTS

The longitudinal characteristic equation is

$$\Delta_{Long} = As'^4 + Bs'^3 + Cs'^2 + Ds' + E$$

where

$$\begin{aligned} A = {} & (m' - X'_{\dot{u}})(m' - Z'_{\dot{w}})(I'_y - M'_{\dot{q}}) - X'_{\dot{w}}M'_{\dot{u}}Z'_{\dot{q}} - Z'_{\dot{u}}M'_{\dot{w}}X'_{\dot{q}} \\ & - (m' - Z'_{\dot{w}})M'_{\dot{u}}X'_{\dot{q}} - Z'_{\dot{u}}X'_{\dot{w}}(I'_y - M'_{\dot{q}}) - M'_{\dot{w}}(m' - X'_{\dot{u}})Z'_{\dot{q}} \; . \end{aligned}$$

$$\begin{aligned} B = {} & - (m' - X'_{\dot{u}})(m' - Z'_{\dot{w}})M'_q - Z'_w(m' - X'_{\dot{u}})(I'_y - M'_{\dot{q}}) \\ & - X'_u(m' - Z'_{\dot{w}})(I'_y - M'_{\dot{q}}) - X'_{\dot{w}}M'_{\dot{u}}(Z'_q + m') - M'_uX'_{\dot{w}}Z'_{\dot{q}} - M'_{\dot{u}}X'_wZ'_{\dot{q}} \\ & - Z'_{\dot{u}}M'_{\dot{w}}X'_q - Z'_{\dot{u}}M'_wZ'_{\dot{q}} - Z'_uM'_{\dot{w}}X'_{\dot{q}} - (m' - Z'_{\dot{w}})M'_{\dot{u}}X'_q - M'_u(m' - Z'_{\dot{w}})X'_{\dot{q}} \\ & + M'_{\dot{u}}Z'_wX'_{\dot{q}} + Z'_{\dot{u}}X'_{\dot{w}}M'_q - Z'_uX'_{\dot{w}}(I'_y - M'_{\dot{q}}) - Z'_{\dot{u}}X'_w(I'_y - M'_{\dot{q}}) \\ & - M'_{\dot{w}}(m' - X'_{\dot{u}})(Z'_q + m') - M'_w(m' - X'_{\dot{u}})Z'_{\dot{q}} + M'_{\dot{w}}X'_uZ'_{\dot{q}} \; . \end{aligned}$$

$$\begin{aligned} C = {} & - (m' - X'_{\dot{u}})(m' - Z'_{\dot{w}})M'_\theta + Z'_w(m' - X'_{\dot{u}})M'_q + X'_u(m' - Z'_{\dot{w}})M'_q \\ & + Z'_wX'_u(I'_y - M'_{\dot{q}}) - X'_{\dot{w}}M'_{\dot{u}}Z'_\theta - M'_uX'_{\dot{w}}(Z'_q + m') - M'_{\dot{u}}X'_w(Z'_q + m') \\ & - Z'_{\dot{u}}M'_{\dot{w}}X'_\theta - Z'_{\dot{u}}M'_wX'_q - Z'_uM'_{\dot{w}}X'_q - Z'_uM'_wX'_{\dot{q}} - (m' - Z'_{\dot{w}})M'_{\dot{u}}X'_\theta \\ & - M'_u(m' - Z'_{\dot{w}})X'_q + M'_{\dot{u}}Z'_wX'_q + Z'_wM'_uX'_{\dot{q}} + Z'_{\dot{u}}X'_{\dot{w}}M'_\theta + Z'_uX'_{\dot{w}}M'_q + Z'_{\dot{u}}X'_wM'_q \\ & - X'_wZ'_u(I'_y - M'_{\dot{q}}) - M'_{\dot{w}}(m' - X'_{\dot{u}})Z'_\theta - M'_w(m' - X'_{\dot{u}})(Z'_q + m') \\ & + M'_{\dot{w}}X'_u(Z'_q + m') + X'_uM'_wZ'_{\dot{q}} - M'_uX'_wZ'_{\dot{q}} \; . \end{aligned}$$

$$\begin{aligned} D = {} & Z'_w M'_\theta (m' - X'_{\dot u}) + X'_u (m' - Z'_{\dot w}) M'_\theta - Z'_w X'_u M'_q - M'_u X'_{\dot w} Z'_\theta - M'_{\dot u} X'_w Z'_\theta \\ & - M'_u X'_w (Z'_q + m') - Z'_{\dot u} M'_w X'_\theta - Z'_u M'_{\dot w} X'_\theta - Z'_u M'_w X'_q - M'_u (m' - Z'_{\dot w}) X'_\theta \\ & + M'_{\dot u} Z'_w X'_\theta + Z'_w M'_u X'_q + Z'_u X'_{\dot w} M'_\theta + Z'_{\dot u} X'_w M'_\theta + X'_w Z'_u M'_q - M'_w (m' - X'_{\dot u}) Z'_\theta \\ & + M'_{\dot w} X'_u Z'_\theta + M'_w X'_u (Z'_q + m') \ . \end{aligned}$$

$$E = - Z'_w X'_u M'_\theta - M'_u X'_w Z'_\theta - Z'_u M'_w X'_\theta + Z'_w M'_u X'_\theta + X'_w Z'_u M'_\theta + X'_u M'_w Z'_\theta \ .$$

The pitch response transfer function is

$$\theta/\delta_s = \frac{N^\theta_{\delta_s}}{\Delta_{\text{Long}}} = \frac{A_\theta s'^2 + B_\theta s' + C_\theta}{\Delta_{\text{Long}}}$$

where

$$\begin{aligned} A_\theta = {} & M'_{\delta_e} (m' - X'_{\dot u})(m' - Z'_{\dot w}) + Z'_{\delta_e} X'_{\dot w} M'_{\dot u} + X'_{\delta_e} Z'_{\dot u} M'_{\dot w} - M'_{\delta_e} X'_{\dot w} Z'_{\dot u} \\ & + X'_{\delta_e} (m' - Z'_{\dot w}) M'_{\dot u} + Z'_{\delta_e} M'_{\dot w} (m' - X'_{\dot u}) \ . \end{aligned}$$

$$\begin{aligned} B_\theta = {} & - M'_{\delta_e} (m' - X'_{\dot u}) Z'_w - M'_{\delta_e} X'_u (m' - Z'_{\dot w}) + Z'_{\delta_e} X'_{\dot w} M'_u + Z'_{\delta_e} X'_w M'_{\dot u} \\ & + X'_{\delta_e} Z'_{\dot u} M'_w + X'_{\delta_e} Z'_u M'_{\dot w} - M'_{\delta_e} X'_{\dot w} Z'_u - M'_{\delta_e} X'_w Z'_{\dot u} + X'_{\delta_e} (m' - Z'_{\dot w}) M'_u \\ & - X'_{\delta_e} Z'_w M'_{\dot u} - Z'_{\delta_e} M'_{\dot w} X'_u + Z'_{\delta_e} M'_w (m' - X'_{\dot u}) \ . \end{aligned}$$

$$C_\theta = M'_{\delta_e} X'_u Z'_w + Z'_{\delta_e} X'_w M'_u + X'_{\delta_e} Z'_u M'_w - M'_{\delta_e} X'_w Z'_u - X'_{\delta_e} Z'_w M'_u - Z'_{\delta_e} M'_w X'_u \ .$$

The vertical velocity transfer function is

$$\frac{w'}{\delta_s} = \frac{N^w_{\delta_s}}{\Delta_{Long}} = \frac{A_w s'^3 + B_w s'^2 + C_w s' + D_w}{\Delta_{Long}}$$

where

$$A_w = Z'_{\delta_e}(m' - X'_{\dot u})(I'_y - M'_{\dot q}) + X'_{\delta_e} M'_{\dot u} Z'_{\dot q} + M'_{\delta_e} Z'_{\dot u} X'_{\dot q} - Z'_{\delta_e} M'_{\dot u} X'_{\dot q}$$

$$+ X'_{\delta_e} Z'_{\dot u}(I'_y - M'_{\dot q}) + M'_{\delta_e}(m' - X'_{\dot u}) Z'_{\dot q} .$$

$$B_w = - Z'_{\delta_e}(m' - X'_{\dot u}) M'_q - Z'_{\delta_e} X'_u (I'_y - M'_{\dot q}) + X'_{\delta_e} M'_u Z'_{\dot q} + X'_{\delta_e} M'_{\dot u}(Z'_q + m')$$

$$+ M'_{\delta_e} Z'_{\dot u} X'_q + M'_{\delta_e} Z'_u X'_{\dot q} - Z'_{\delta_e} M'_{\dot u} X'_q - Z'_{\delta_e} M'_u X'_{\dot q} - X'_{\delta_e} Z'_{\dot u} M'_q$$

$$+ X'_{\delta_e} Z'_u (I'_y - M'_{\dot q}) + M'_{\delta_e}(m' - X'_{\dot u})(Z'_q + m') - M'_{\delta_e} X'_u Z'_{\dot q} .$$

$$C_w = - Z'_{\delta_e}(m' - X'_{\dot u}) M'_\theta + Z'_{\delta_e} X'_u M'_q + X'_{\delta_e} M'_{\dot u} Z'_\theta + X'_{\delta_e} M'_u (Z'_q + m') + M'_{\delta_e} Z'_{\dot u} X'_\theta$$

$$+ M'_{\delta_e} Z'_u Z'_q - Z'_{\delta_e} M'_{\dot u} X'_\theta - Z'_{\delta_e} M'_u X'_q - X'_{\delta_e} Z'_{\dot u} M'_\theta - X'_{\delta_e} Z'_u M'_q + M'_{\delta_e}(m' - X'_{\dot u}) Z'_\theta$$

$$- M'_{\delta_e} X'_u (Z'_q + m').$$

$$D_w = Z'_{\delta_e} X'_u M'_\theta + X'_{\delta_e} M'_u Z'_\theta + M'_{\delta_e} Z'_u X'_\theta - Z'_{\delta_e} M'_u X'_\theta - X'_{\delta_e} Z'_u M'_\theta - M'_{\delta_e} X'_u Z'_\theta .$$

The forward speed transfer function is

$$\frac{u'}{\delta_s} = \frac{N^u_{\delta_s}}{\Delta_{Long}} = \frac{A_u s'^3 + B_u s'^2 + C_u s' + D_u}{\Delta_{Long}}$$

where

$$A_u = X'_{\delta_e}(m' - Z'_{\dot w})(I'_y - M'_{\dot q}) + M'_{\delta_e} X'_{\dot w} Z'_{\dot q} + Z'_{\delta_e} M'_{\dot w} X'_{\dot q} + M'_{\delta_e}(m' - Z'_{\dot w}) X'_{\dot q}$$

$$+ Z'_{\delta_e} X'_{\dot w}(I'_y - M'_{\dot q}) - X'_{\delta_e} M'_{\dot w} Z'_{\dot q} .$$

$$
\begin{aligned}
B_u = & - X'_{\delta_e}(m' - Z'_{\dot w})M'_q - X'_{\delta_e}(I'_y - M'_{\dot q})Z'_w + M'_{\delta_e} X'_{\dot w}(Z'_q + m') + M'_{\delta_e} X'_w Z'_{\dot q} \\
& + Z'_{\delta_e} M'_{\dot w} X'_q + Z'_{\delta_e} M'_w X'_{\dot q} + M'_{\delta_e}(m' - Z'_{\dot w})X'_q - M'_{\delta_e} Z'_w X'_{\dot q} - Z'_{\delta_e} X'_{\dot w} M'_q \\
& + Z'_{\delta_e} X'_w (I'_y - M'_{\dot q}) - X'_{\delta_e} M'_{\dot w}(Z'_q + m') - X'_{\delta_e} M'_w Z'_{\dot q} .
\end{aligned}
$$

$$
\begin{aligned}
C_u = & - X'_{\delta_e}(m' - Z'_{\dot w})M'_\theta + X'_{\delta_e} Z'_w M'_q + M'_{\delta_e} X'_{\dot w} Z'_\theta + M'_{\delta_e} X'_w (Z'_q + m') + Z'_{\delta_e} M'_{\dot w} X'_\theta \\
& + Z'_{\delta_e} M'_w X'_q + M'_{\delta_e}(m' - Z'_{\dot w})X'_\theta - M'_{\delta_e} Z'_w X'_q - Z'_{\delta_e} X'_{\dot w} M'_\theta - Z'_{\delta_e} X'_w M'_q \\
& - X'_{\delta_e} M'_{\dot w} Z'_\theta - X'_{\delta_e} M'_w (Z'_q + m') .
\end{aligned}
$$

$$
D_u = X'_{\delta_e} Z'_w M'_\theta + M'_{\delta_e} X'_w Z'_\theta + Z'_{\delta_e} M'_w X'_\theta - M'_{\delta_e} Z'_w X'_\theta - Z'_{\delta_e} X'_w M'_\theta - X'_{\delta_e} M'_w Z'_\theta .
$$

EXPRESSIONS FOR THE LATERAL TRANSFER FUNCTION COEFFICIENTS

The lateral characteristic equation is

$$\Delta_{Lat} = s'(As'^4 + Bs'^3 + Cs'^2 + Ds' + E)$$

where

$$\begin{aligned}
A = {} & (m' - Y'_{\dot v})(I'_z - N'_{\dot r})(I'_x - K'_{\dot p}) + N'_{\dot v}(- I'_{xz} - K'_{\dot r})Y'_{\dot p} \\
& + Y'_{\dot r}K'_{\dot v}(- I'_{xz} - N'_{\dot p}) - N'_{\dot v}Y'_{\dot r}(I'_x - K'_{\dot p}) - K'_{\dot v}(I'_z - N'_{\dot r})Y'_{\dot p} \\
& - (m' - Y'_{\dot v})(- I'_{xz} - N'_{\dot p})(- I'_{xz} - K'_{\dot r}) \ .
\end{aligned}$$

$$\begin{aligned}
B = {} & - (m' - Y'_{\dot v})(I'_z - N'_{\dot r})K'_p - N'_r(m' - Y'_{\dot v})(I'_x - K'_{\dot p}) \\
& - Y'_v(I'_z - N'_{\dot r})(I'_x - K'_{\dot p}) + N'_{\dot v}(- I'_{xz} - K'_{\dot r})Y'_p - N'_{\dot v}K'_r Y'_{\dot p} \\
& + N'_v(- I'_{xz} - K'_{\dot r})Y'_{\dot p} - Y'_{\dot r}K'_{\dot v}N'_p + Y'_{\dot r}K'_v(- I'_{xz} - N'_{\dot p}) \\
& - (m' - Y'_r)K'_{\dot v}(- I'_{xz} - N'_{\dot p}) + N'_{\dot v}Y'_{\dot r}K'_p + N'_{\dot v}(m' - Y'_r)(I'_x - K'_{\dot p}) \\
& - N'_v Y'_{\dot r}(I'_x - K'_{\dot p}) - K'_{\dot v}(I'_z - N'_{\dot r})Y'_p + K'_{\dot v}N'_r Y'_{\dot p} - (I'_z - N'_{\dot r})K'_v Y'_{\dot p} \\
& + (m' - Y'_{\dot v})(- I'_{xz} - K'_{\dot r})N'_p + (m' - Y'_{\dot v})K'_r(- I'_{xz} - N'_{\dot p}) \\
& + Y'_v(- I'_{xz} - K'_{\dot r})(- I'_{xz} - N'_{\dot p}) \ .
\end{aligned}$$

$$C = N'_r(m' - Y'_{\dot v})K'_p - (m' - Y'_{\dot v})(I'_z - N'_{\dot r})K'_\phi + Y'_v(I'_z - N'_{\dot r})K'_p$$

$$+ N'_r Y'_v(I'_x - K'_{\dot p}) + N'_{\dot v}(- I'_{xz} - K'_{\dot r})Y'_\phi - N'_{\dot v}K'_r Y'_p + N'_v(- I'_{xz} - K'_{\dot r})Y'_p$$

$$- N'_v K'_r Y'_{\dot p} - Y'_{\dot r}K'_{\dot v}N'_\phi - Y'_{\dot r}K'_v N'_p + (m' - Y'_r)K'_{\dot v}N'_p$$

$$- (m' - Y'_r)K'_v(- I'_{xz} - N'_{\dot p}) + N'_{\dot v}Y'_{\dot r}K'_\phi - N'_{\dot v}(m' - Y'_r)K'_p + N'_v Y'_{\dot r}K'_p$$

$$+ N'_v(m' - Y'_r)(I'_x - K'_{\dot p}) - K'_{\dot v}(I'_z - N'_{\dot r})Y'_\phi + K'_{\dot v}N'_r Y'_p - (I'_z - N'_{\dot r})K'_v Y'_p$$

$$+ K'_v N'_r Y'_{\dot p} + (m' - Y'_{\dot v})(- I'_{xz} - K'_{\dot r})N'_\phi - (m' - Y'_{\dot v})K'_r N'_p$$

$$- Y'_v(- I'_{xz} - K'_{\dot r})N'_p - K'_r Y'_v(- I'_{xz} - N'_{\dot p}) .$$

$$D = N'_r(m' - Y'_{\dot v})K'_\phi + Y'_v(I'_z - N'_{\dot r})K'_\phi - N'_r Y'_v K'_p - N'_{\dot v}K'_r Y'_\phi$$

$$+ N'_v(- I'_{xz} - K'_{\dot r})Y'_\phi - N'_v K'_r Y'_p - Y'_{\dot r}K'_v N'_\phi + (m' - Y'_r)K'_{\dot v}N'_\phi$$

$$+ (m' - Y'_r)K'_v N'_p - N'_{\dot v}(m' - Y'_r)K'_\phi + N'_v Y'_{\dot r}K'_\phi - N'_v(m' - Y'_r)K'_p + K'_{\dot v}N'_r Y'_\phi$$

$$- (I'_z - N'_{\dot r})K'_v Y'_\phi + K'_v N'_r Y'_p - (m' - Y'_{\dot v})K'_r N'_\phi - Y'_v(- I'_{xz} - K'_{\dot r})N'_\phi$$

$$+ K'_r Y'_v N'_p .$$

$$E = - N'_r Y'_v K'_\phi - N'_v K'_r Y'_\phi + (m' - Y'_r)K'_v N'_\phi - N'_v(m' - Y'_r)K'_\phi + K'_v N'_r Y'_\phi + K'_r Y'_v N'_\phi .$$

The sideslip transfer function is

$$\frac{\beta}{\delta_R} = \frac{N^{\beta}_{\delta_R}}{\Delta_{Lat}} = \frac{s'(A_\beta s'^3 + B_\beta s'^2 + C_\beta s' + D_\beta)}{\Delta_{Lat}}$$

where

$$A_\beta = Y'_{\delta_r}(I'_z - N'_{\dot{r}})(I'_x - K'_{\dot{p}}) - N'_{\delta_r}(-I'_{xz} - K'_{\dot{r}})Y'_{\dot{p}} - K'_{\delta_r}Y'_{\dot{r}}(-I'_{xz} - N'_{\dot{p}})$$

$$+ N'_{\delta_r}Y'_{\dot{r}}(I'_x - K'_{\dot{p}}) + K'_{\delta_r}(I'_z - N'_{\dot{r}})Y'_{\dot{p}} - Y'_{\delta_r}(-I'_{xz} - K'_{\dot{r}})(-I'_{xz} - N'_{\dot{p}}) \ .$$

$$B_\beta = -Y'_{\delta_r}(I'_z - N'_{\dot{r}})K'_p - Y'_{\delta_r}N'_r(I'_x - K'_{\dot{p}}) - N'_{\delta_r}(-I'_{xz} - K'_{\dot{r}})Y'_p + N'_{\delta_r}K'_rY'_{\dot{p}}$$

$$+ K'_{\delta_r}Y'_{\dot{r}}N'_p + K'_{\delta_r}(m' - Y'_r)(-I'_{xz} - N'_{\dot{p}}) - N'_{\delta_r}Y'_{\dot{r}}K'_p$$

$$- (m' - Y'_r)N'_{\delta_r}(I'_x - K'_{\dot{p}}) + K'_{\delta_r}(I'_z - N'_{\dot{r}})Y'_p - K'_{\delta_r}N'_rY'_{\dot{p}}$$

$$+ Y'_{\delta_r}(-I'_{xz} - K'_{\dot{r}})N'_p + Y'_{\delta_r}K'_r(-I'_{xz} - N'_{\dot{p}}) \ .$$

$$C_\beta = -Y'_{\delta_r}(I'_z - N'_{\dot{r}})K'_\phi + Y'_{\delta_r}N'_rK'_p - N'_{\delta_r}(-I'_{xz} - K'_{\dot{r}})Y'_\phi + N'_{\delta_r}K'_rY'_p$$

$$+ K'_{\delta_r}Y'_{\dot{r}}N'_\phi - K'_{\delta_r}(m' - Y'_r)N'_p - N'_{\delta_r}Y'_{\dot{r}}K'_\phi + (m' - Y'_r)N'_{\delta_r}K'_p$$

$$+ K'_{\delta_r}(I'_z - N'_{\dot{r}})Y'_\phi - N'_rK'_{\delta_r}Y'_p + Y'_{\delta_r}(-I'_{xz} - K'_{\dot{r}})N'_\phi - Y'_{\delta_r}K'_rN'_p \ .$$

$$D_\beta = Y'_{\delta_r}N'_rK'_\phi + N'_{\delta_r}K'_rY'_\phi - K'_{\delta_r}(m' - Y'_r)N'_\phi + (m' - Y'_r)N'_{\delta_r}K'_\phi - N'_rK'_{\delta_r}Y'_\phi$$

$$- Y'_{\delta_r}K'_rN'_\phi \ .$$

The roll transfer function is

$$\frac{\phi}{\delta_R} = \frac{N^{\phi}_{\delta_R}}{\Delta_{Lat}} = \frac{s'(A_\phi s'^2 + B_\phi s' + C_\phi)}{\Delta_{Lat}}$$

where

$$A_\phi = K'_{\delta_r}(m' - Y'_{\dot v})(I'_z - N'_{\dot r}) - Y'_{\delta_r} N'_{\dot v}(- I'_{xz} - K'_{\dot r}) + N'_{\delta_r} K'_{\dot v} Y'_{\dot r}$$

$$- K'_{\delta_r} N'_{\dot v} Y'_{\dot r} + Y'_{\delta_r} K'_{\dot v}(I'_z - N'_{\dot r}) - N'_{\delta_r}(m' - Y'_{\dot v})(- I'_{xz} - K'_{\dot r}) .$$

$$B_\phi = - K'_{\delta_r}(m' - Y'_{\dot v})N'_r - K'_{\delta_r} Y'_v(I'_z - N'_{\dot r}) + Y'_{\delta_r} N'_{\dot v} K'_r - Y'_{\delta_r} N'_v(- I'_{xz} - K'_{\dot r})$$

$$- N'_{\delta_r} K'_{\dot v}(m' - Y'_r) + N'_{\delta_r} K'_v Y'_{\dot r} + K'_{\delta_r} N'_{\dot v}(m' - Y'_r) - K'_{\delta_r} N'_v Y'_{\dot r} - Y'_{\delta_r} K'_{\dot v} N'_r$$

$$+ Y'_{\delta_r} K'_v(I'_z - N'_{\dot r}) + N'_{\delta_r}(m' - Y'_{\dot v})K'_r + N'_{\delta_r} Y'_v(- I'_{xz} - K'_{\dot r}) .$$

$$C_\phi = K'_{\delta_r} Y'_v N'_r + Y'_{\delta_r} N'_v K'_r - N'_{\delta_r} K'_v(m' - Y'_r) + K'_{\delta_r} N'_v(m' - Y'_r) - Y'_{\delta_r} K'_v N'_r$$

$$- N'_{\delta_r} Y'_v K'_r .$$

The yaw transfer function is

$$\frac{\psi}{\delta_R} = \frac{N^{\psi}_{\delta_R}}{\Delta_{Lat}} = \frac{A_\psi s'^3 + B_\psi s'^2 + C_\psi s' + D_\psi}{\Delta_{Lat}}$$

where

$$A_\psi = N'_{\delta_r}(m' - Y'_{\dot v})(I'_x - K'_{\dot p}) + K'_{\delta_r} N'_{\dot v} Y'_{\dot p} - Y'_{\delta_r} K'_{\dot v}(- I'_{xz} - N'_{\dot p})$$

$$+ Y'_{\delta_r} N'_{\dot v}(I'_x - K'_{\dot p}) - N'_{\delta_r} K'_{\dot v} Y'_{\dot p} - K'_{\delta_r}(m' - Y'_{\dot v})(- I'_{xz} - N'_{\dot p}) .$$

$$B_\psi = - N'_{\delta_r}(m' - Y'_{\dot v})K'_p - N'_{\delta_r} Y'_v(I'_x - K'_{\dot p}) + K'_{\delta_r} N'_{\dot v} Y'_p + K'_{\delta_r} N'_v Y'_{\dot p}$$

$$+ Y'_{\delta_r} K'_{\dot v} N'_p - Y'_{\delta_r} K'_v(- I'_{xz} - N'_{\dot p}) - Y'_{\delta_r} N'_{\dot v} K'_p + Y'_{\delta_r} N'_v(I'_x - K'_{\dot p})$$

$$- N'_{\delta_r} K'_{\dot v} Y'_p - N'_{\delta_r} K'_v Y'_{\dot p} + K'_{\delta_r}(m' - Y'_{\dot v})N'_p + K'_{\delta_r} Y'_v(- I'_{xz} - N'_{\dot p}) .$$

$$
\begin{aligned}
C_\psi = & - N'_{\delta_r}(m' - Y'_{\dot{v}})K'_\phi + N'_{\delta_r}Y'_v K'_p + K'_{\delta_r}N'_{\dot{v}}Y'_\phi + K'_{\delta_r}N'_v Y'_p + Y'_{\delta_r}K'_{\dot{v}}N'_\phi \\
& + Y'_{\delta_r}K'_v N'_p - Y'_{\delta_r}N'_{\dot{v}}K'_\phi - Y'_{\delta_r}N'_v K'_p - N'_{\delta_r}K'_{\dot{v}}Y'_\phi - N'_{\delta_r}K'_v Y'_p + K'_{\delta_r}(m' - Y'_{\dot{v}})N'_\phi \\
& - K'_{\delta_r}Y'_v N'_p \ .
\end{aligned}
$$

$$
D_\psi = N'_{\delta_r}Y'_v K'_\phi + K'_{\delta_r}N'_v Y'_\phi + Y'_{\delta_r}K'_v N'_\phi - Y'_{\delta_r}N'_v K'_\phi - N'_{\delta_r}K'_v Y'_\phi - K'_{\delta_r}Y'_v N'_\phi \ .
$$

APPENDIX C

VERTLIN.FORT

```
C
      REAL A,B,C,D,E
      INTEGER NDEG,IER
      READ(4,*) WT,XUD,ZWD,RIY,XL,U
      READ(4,*) RMUD,ZQD,XWD,RMQD,RMWD
      READ(4,*) XQD,ZUD,RMQ,ZW,XU,ZQ
      READ(4,*) RMU,XW,XQ,ZU,RMTHET
      READ(4,*) ZTHETA,XTHETA,RMW,XDE,ZDE,RMDE
C23456
C
      REAL A1(5),A2(3),A3(4),A4(4)
      COMPLEX Z1(4),Z2(2),Z3(3),Z4(3)
C
      WRITE(6,15)
 15   FORMAT(1X,'IN WHAT FORMAT DO YOU WANT THE ANSWER?',/,
     1 '    DIMENSIONAL        WRITE 1',/,
     2 '    NONDIMENSIONAL     WRITE 0')
      READ(7,20) J
      IF (J) 2,2,1
 1    WRITE(5,4)
      WRITE(6,4)
 4    FORMAT(1X,'OUTPUT VALUES ARE IN DIMENSIONAL FORM')
      GO TO 3
C
C
  2   WRITE(5,5)
      WRITE(6,5)
 5    FORMAT(1X,'OUTPUT VALUES ARE IN NONDIMENSIONAL FORM')
 3    CONTINUE
 20   FORMAT(I1)
C
C
C
      XLU=XL/U
      XLU2=XLU*XLU
      XLU3=XLU2*XLU
      XLU4=XLU3*XLU
      XLU5=XLU4*XLU
C
C

C
      A= (WT-XUD)*(WT-ZWD)*(RIY-RMQD)-XWD*RMUD*ZQD-ZUD*RMWD*XQD
     1   -(WT-ZWD)*RMUD*XQD-ZUD*XWD*(RIY-RMQD)-RMWD*(WT-XUD)*ZQD
C234567
C2345678911234567892123456789312345678941234567895123456789612345
      B= -(WT-XUD)*(WT-ZWD)*RMQ-ZW*(WT-XUD)*(RIY-RMQD)
     1   -XU*(WT-ZWD)*(RIY-RMQD)-XWD*RMUD*(ZQ+WT)-RMU*XWD*ZQD
     2   -RMUD*XW*ZQD-ZUD*RMWD*XQ-ZUD*RMW*ZQD-ZU*RMWD*XQD-(WT-ZWD)*
     3    RMUD*XQ-RMU*(WT-ZWD)*XQD+RMUD*ZW*XQD+ZUD*XWD*RMQ
     4   -ZU*XWD*(RIY-RMQD)-ZUD*XW*(RIY-RMQD)-RMWD*(WT-XUD)*(ZQ+WT)
     5   -RMW*(WT-XUD)*ZQD+RMWD*XU*ZQD
C
C
      C=  -(WT-XUD)*(WT-ZWD)*RMTHET+ZW*(WT-XUD)*RMQ+XU*(WT-ZWD)*
     1   RMQ+ZW*XU*(RIY-RMQD)-XWD*RMUD*ZTHETA-RMU*XWD*(ZQ+WT)-
```

VERTLIN.FORT (cont.)

```
     2     RMUD*XW*(ZQ+WT)-ZUD*RMWD*XTHETA-ZUD*RMW*XQ-ZU*RMWD*XQ-ZU*RMW*
     3     XQD-(WT-ZWD)*RMUD*XTHETA-RMU*(WT-ZWD)*XQ+RMUD*ZW*XQ+
     4     ZW*RMU*XQD+ZUD*XWD*RMTHET+ZU*XWD*RMQ+ZUD*XW*RMQ-
     5     XW*ZU*(RIY-RMQD)-RMWD*(WT-XUD)*ZTHETA-RMW*(WT-XUD)*
     6     (ZQ+WT)+RMWD*XU*(ZQ+WT)+XU*RMW*ZQD-RMU*XW*ZQD
C
C234567
      D=    ZW*RMTHET*(WT-XUD)+XU*(WT-ZWD)*RMTHET-ZW*XU*RMQ-RMU*
     1     XWD*ZTHETA-RMUD*XW*ZTHETA-RMU*XW*(ZQ+WT)-ZUD*RMW*XTHETA
     2     -ZU*RMWD*XTHETA-ZU*RMW*XQ-RMU*(WT-ZWD)*XTHETA+RMUD*ZW*
     3     XTHETA+ZW*RMU*XQ+ZU*XWD*MTHETA+ZUD*XW*RMTHET+XW*ZU*RMQ
     4     -RMW*(WT-XUD)*ZTHETA+RMWD*XU*ZTHETA+RMW*XU*(ZQ+WT)
C23456789112345678921234567893123456789412345678951234567896123 45
C
      E=   -ZW*XU*RMTHET-RMU*XW*ZTHETA-ZU*RMW*XTHETA+ZW*RMU*XTHETA
     1     +XW*ZU*RMTHET+XU*RMW*ZTHETA
C
C
      IF(J) 22,22,21
   21 A=A*XLU4
      B=B*XLU3
      C=C*XLU2
      D=D*XLU
C
C
C
   22 WRITE(5,50)
      WRITE(6,50)
   50 FORMAT(1X,'COEFFS. OF CHARACTERISTIC EQUATION',/)
       WRITE(5,51)
       WRITE(6,51)
   51  FORMAT(7X,'A',14X,'B',14X,'C',14X,'D',14X,'E')
      WRITE(5,100)A,B,C,D,E
      WRITE(6,100)A,B,C,D,E
  100 FORMAT(1X,5E14.6,/)
      NDEG=4
      A1(1)=A
      A1(2)=B
      A1(3)=C
      A1(4)=D
      A1(5)=E
C
C
      CALL ZPOLR(A1,NDEG,Z1,IER)
C23456789
C
C
      WRITE(5,200)
      WRITE(6,200)
  200 FORMAT(1X,'ROOTS OF CHARACTERISTIC EQUATION',/)
      WRITE(5,205)
      WRITE(6,205)
  205 FORMAT(10X,'REAL',11X,'IMAGINARY')
C
      WRITE(5,201) Z1
      WRITE(6,201) Z1
  201 FORMAT(1X,2F16.6,//)
C
C
C  PITCH RESPONSE TRANSFER FUNCTION NUMERATOR POLYNOMIAL
```

VERTLIN.FORT (cont.)

```
C
C
C
      ATHETA =  RMDE*(WT-XUD)*(WT-ZWD)+ZDE*XWD*RMUD+XDE*ZUD*RMWD
     1   -RMDE*XWD*ZUD+XDE*(WT-ZWD)*RMUD+ZDE*MWD*(WT-XUD)
C
C234567
      BTHETA =  -RMDE*(WT-XUD)*ZW-RMDE*XU*(WT-ZWD)+ZDE*XWD*RMU+
     1  ZDE*XW*RMUD+XDE*ZUD*RMW+XDE*ZU*RMD-RMDE*XWD*ZU
     2  -RMDE*XW*ZUD+XDE*(WT-ZWD)*RMU-XDE*ZW*RMUD
     3  -ZDE*RMWD*XU+ZDE*RMW*(WT-XUD)
C
C
      CTHETA = RMDE*XU*ZW+ZDE*XW*RMU+XDE*ZU*RMW-RMDE*XW*ZU
     1  -XDE*ZW*RMU-ZDE*RMW*XU
C2345678
C
      IF(J) 32,32,31
  31  ATHETA =ATHETA*XLU2
      BTHETA =BTHETA*XLU
C
C
  32  WRITE(5,52)
      WRITE(6,52)
  52  FORMAT(1X,'PITCH POLYNOMIAL COEFFICIENTS'/)
      WRITE(5,53)
      WRITE(6,53)
  53  FORMAT(7X,'ATHETA',9X,'BTHETA',9X,'CTHETA')
      WRITE(5,100) ATHETA,BTHETA,CTHETA
      WRITE(6,100) ATHETA,BTHETA,CTHETA
C
      NDEG=2
      A2(1)=ATHETA
      A2(2)=BTHETA
      A2(3)=CTHETA
C
C
      CALL ZPOLR(A2,NDEG,Z2,IER)
C
C
      WRITE(5,54)
      WRITE(6,54)
  54  FORMAT(1X,'ROOTS')
      WRITE(5,205)
      WRITE(6,205)
      WRITE(5,201) Z2
      WRITE(6,201) Z2
C
C  VERTICAL VELOCITY TRANSFER FUNCTION NUMERATOR POLYNOMIAL
C
       AW = ZDE*(WT-XUD)*(RIY-RMQD)+XDE*RMUD*ZQD+RMDE*ZUD*XQD-ZDE*RMUD
     1     *XQD+XDE*ZUD*(RIY-RMQD)+MDE*(WT-XUD)*ZQD
C234567
C
       BW = -ZDE*(WT-XUD)*RMQ-ZDE*XU*(RIY-RMQD)+XDE*RMU*ZQD+XDE*RMUD
     1     *(ZQ+WT)+MDE*ZUD*XQ+RMDE*ZU*XQD-ZDE*RMUD*XQ-ZDE*RMU*XQD
     2     -XDE*ZUD*RMQ+XDE*ZU*(RIY-RMQD)+RMDE*(WT-XUD)*(ZQ+WT)
     3     -RMDE*XU*ZQD
C23456789112345678921234567893123456789412345678951234567896123456
      CW = -ZDE*(WT-XUD)*RMTHET+ZDE*XU*RMQ+XDE*RMUD*ZTHETA+XDE*RMU
```

VERTLIN.FORT (cont.)

```
     1   *(ZQ+WT)+RMDE*ZUD*XTHETA+RMDE*ZU*ZQ-ZDE*RMUD*XTHETA
     2   -ZDE*RMU*XQ-XDE*ZUD*RMTHET-XDE*ZU*RMQ+RMDE*(WT-XUD)
     3   *ZTHETA-RMDE*XU*(ZQ+WT)
C
C
         DW = ZDE*XU*RMTHET+XDE*RMU*ZTHETA+RMDE*ZU*XTHETA
     1     -ZDE*RMU*XTHETA-XDE*ZU*RMTHET-RMDE*XU*ZTHETA
C
C
C
         IF(J) 42,42,41
  41     AW=AW*XLU3
         BW=BW*XLU2
         CW=CW*XLU
C
C
  42     WRITE(5,105)
         WRITE(6,105)
  105    FORMAT(1X,'VERTICAL VELOCITY COEFFICIENTS',/)
         WRITE(5,106)
         WRITE(6,106)
  106    FORMAT(7X,'AW',9X,'BW',9X,'CW',9X,'DW')
         WRITE(5,100) AW,BW,CW,DW
         WRITE(6,100) AW,BW,CW,DW
C
C
         NDEG=3
         A3(1)=AW
         A3(2)=BW
         A3(3)=CW
         A3(4)=DW
C
C
         CALL ZPOLR(A3,NDEG,Z3,IER)
C
C
         WRITE(5,202)
         WRITE(6,202)
  202    FORMAT(1X,'ROOTS')
         WRITE(5,205)
         WRITE(6,205)
         WRITE(5,201) Z3
         WRITE(6,201) Z3
C
C
C23456789
        AU = XDE*(WT-ZWD)*(RIY-RMQD)+RMDE*XWD*ZQD+ZDE*RMWD*XQD+RMDE*
     1   (WT-ZWD)*XQD+ZDE*XWD*(RIY-RMQD)-XDE*RMWD*ZQD
C
C
        BU = -XDE*(WT-ZWD)*RMQ-XDE*(RIY-RMQD)*ZW+RMDE*XWD*(ZQ+WT)
     1   +RMDE*XW*ZQD+ZDE*RMWD*XQ+ZDE*RMW*XQD+RMDE*(WT-ZWD)*XQ
     2   -RMDE*ZW*XQD-ZDE*XWD*RMQ+ZDE*XW*(RIY-RMQD)-XDE*RMWD
     3   *(ZQ+WT)-XDE*RMW*ZQD
C
C
C
         CU = -XDE*(WT-ZWD)*RMTHET+XDE*ZW*RMQ+RMDE*XWD*ZTHETA+RMDE*XW*
     1    (ZQ+WT)+ZDE*RMWD*XTHETA+ZDE*RMW*XQ+RMDE*(WT-ZWD)*XTHETA
     2    -RMDE*ZW*XQ-ZDE*XWD*RMTHET-ZDE*XW*RMQ-XDE*RMWD*ZTHETA
```

VERTLIN.FORT (cont.)

```
     3    -XDE*RMW*(ZQ+WT)
C
C
           DU = XDE*ZW*RMTHET+RMDE*XW*ZTHETA+ZDE*RMW*XTHETA
     1       - RMDE*ZW*XTHETA-ZDE*XW*RMTHET-XDE*RMW*ZTHETA
C
C
C
         IF(J) 62,62,61
  61     AU = AU*XLU3
         BU = BU*XLU2
         CU = CU*XLU
C
C
  62     WRITE(5,60)
         WRITE(6,60)
  60     FORMAT(1X,'FORWARD SPEED POLYNOMIAL COEFFICIENTS',/)
         WRITE(5,70)
         WRITE(6,70)
  70     FORMAT(7X,'AU',11X,'BU',11X,'CU',11X,'DU')
         WRITE(5,100) AU,BU,CU,DU
         WRITE(6,100) AU,BU,CU,DU
C
C
         NDEG=3
         A4(1)=AU
         A4(2)=BU
         A4(3)=CU
         A4(4)=DU
C
C
         CALL ZPOLR(A4,NDEG,Z4,IER)
C
C
         WRITE(5,202)
         WRITE(6,202)
         WRITE(5,205)
         WRITE(6,205)
         WRITE(5,201) Z4
         WRITE(6,201) Z4
C
C
         STOP
         END
```

HORLIN.FORT

```
C
      REAL A,B,C,D,E
      INTEGER NDEG,IER
      READ(4,*)WT,RIX,RIY,RIZ,XL,U
      READ(4,*)YV,YR,YPHI,YVD,YPD,YRD,YDR,YP
      READ(4,*)RKV,RKP,RKR,RKPHI,RKVD,RKPD,RKRD,RKDR
      READ(4,*)RNV,RNR,RNVD,RNRD,RNDR,RNPD,RNP,RNPHI
      REAL A1(5),A3(3),A2(4),A4(4)
      COMPLEX Z1(4),Z3(2),Z2(3),Z4(3)
      WRITE(6,15)
 15    FORMAT(1X,'IN WHAT FORMAT DO YOU WANT THE ANSWER?',/,
     1'  DIMENSIONAL        WRITE 1',/,
     2'  NONDIMENSIONAL     WRITE 0')
      READ(7,20)J
      IF(J)2,2,1
 1     WRITE(5,4)
      WRITE(6,4)
 4     FORMAT(1X,'OUTPUT VALUES ARE IN DIMENSIONAL FORM')
      GO TO 3
 2     WRITE(5,5)
      WRITE(6,5)
 5    FORMAT(1X,'OUTPUT VALUES ARE IN NONDIMENSIONAL FORM')
  3   CONTINUE
 20    FORMAT(I1)
C
C
      XLU = XL/U
      XLU2 = XLU*XLU
      XLU3 = XLU2*XLU
      XLU4 = XLU3*XLU
      XLU5 = XLU4*XLU
C
C
      A=(WT-YVD)*(RIZ-RNRD)*(RIX-RKPD) + RNVD*(-RKRD)*YPD
     1 +YRD*RKVD*(-RNPD)-RNVD*YRD*(RIX-RKPD)-RKVD*(RIZ-RNRD)*YPD
     2 -(WT-YVD)*(-RNPD)*(-RKRD)
C
C
      B = -(WT-YVD)*(RIZ-RNRD)*RKP - RNR*(WT-YVD)*(RIX-RKPD)
     1 -YV*(RIZ-RNRD)*(RIX-RKPD)+RNVD*(-RKRD)*YP-RNVD*RKR*YPD
     2 +RNV*(-RKRD)*YPD -YRD*RKVD*RNP +YRD*RKV*(-RNPD)
     3 -(WT-YR)*RKVD*(-RNPD) + RNVD*YRD*RKP + RNVD*(WT-YR)*(RIX-RKPD)
     4 -RNV*YRD*(RIX-RKPD) - RKVD*(RIZ-RNRD)*YP +RKVD*RNR*YPD-(RIZ-RNRD)
     5 *RKV*YPD
     6 +(WT-YVD)*(-RKRD)*RNP + (WT-YVD)*RKR*(-RNPD)
     7 + YV*(-RKRD)*(-RNPD)
C
C
C
      C = RNR*(WT-YVD)*RKP -(WT-YVD)*(RIZ-RNRD)*RKPHI+YV*(RIZ-RNRD)*RKP
     1 +RNR*YV*(RIX-RKPD)+RNVD*(-RKRD)*YPHI-RNVD*RKR*YP +RNV*(-RKRD)
     2 *YP -RNV*RKR*YPD -YRD*RKVD*RNPHI -YRD*RKV*RNP +(WT-YR)*RKVD*RNP
     3 - (WT-YR)*RKV*(-RNPD) + RNVD*YRD*RKPHI - RNVD*(WT-YR)*RKP+
     4 RNV*YRD*RKP + RNV*(WT-YR)*(RIX-RKPD) - RKVD*(RIZ-RNRD)*YPHI+
     5 RKVD*RNR*YP - (RIZ-RNRD)*RKV*YP +RKV*RNR*YPD +(WT-YVD)*(-RKRD)
     6 *RNPHI -(WT-YVD)*RKR*RNP - YV*(-RKRD)*RNP - RKR*YV*(-RNPD)
```

HORLIN.FORT (cont.)

```
C
C
      D = RNR*(WT-YVD)*RKPHI + YV*(RIZ-RNRD)*RKPHI - RNR*YV*RKP
     1 -RNVD*RKR*YPHI + RNV*(-RKRD)*YPHI - RNV*RKR*YP - YRD*RKV*RNPHI
     2 +(WT-YR)*RKVD*RNPHI + (WT-YR)*RKV*RNP -RNVD*(WT-YR)*RKPHI+
     3 RNV*YRD*RKPHI -RNV*(WT-YR)*RKP +RKVD*RNR*YPHI
     4 - (RIZ-RNRD)*RKV*YPHI + RKV*RNR*YP - (WT-YVD)*RKR*RNPHI -
     5 YV*(-RKRD)*RNPHI + RKR*YV*RNP
      E = -RNR*YV*RKPHI - RNV*RKR*YPHI + (WT-YR)*RKV*RNPHI
     1 -RNV*(WT-YR)*RKPHI + RKV*RNR*YPHI + RKR*YV*NPHI
      IF(J)22,22,21
 21    A = A*XLU5
      B = B*XLU4
      C = C*XLU3
      D = D*XLU2
      E = E*XLU
C
C
 22    WRITE(5,50)
      WRITE(6,50)
 50    FORMAT(1X,'COEFFS. OF CHARACTERISTIC EQUATION',/)
      WRITE(5,51)
      WRITE(6,51)
 51    FORMAT(7X,'A',14X,'B',14X,'C',14X,'D',14X,'E')
      WRITE(6,100) A,B,C,D,E
      WRITE(5,100) A,B,C,D,E
 100   FORMAT (1X,5E14.6,/)
      NDEG = 4
      A1(1) = A
      A1(2) = B
      A1(3) = C
      A1(4) = D
      A1(5) = E
      CALL ZPOLR(A1,NDEG,Z1,IER)
      WRITE(5,200)
      WRITE(6,200)
 200   FORMAT(1X,'ROOTS OF CHARACTERISTIC EQUATION',/)
      WRITE(5,205)
      WRITE(6,205)
 205   FORMAT(10X,'REAL',11X,'IMAGINARY')
      WRITE(5,201)Z1
      WRITE(6,201)Z1
 201   FORMAT(1X,2F16.6,//)
C
C
C
      ABETA = YDR*(RIZ-RNRD)*(RIX-RKPD) - RNDR*(-RKRD)*YPD
     1 - RKDR*YRD*(-RNPD) + RNDR*YRD*(RIX-RKPD)
     2 + RKDR*(RIZ-RNRD)*YPD -YDR*(-RKRD)*(-RNPD)
C
C
C
      BBETA = -YDR*(RIZ-RNRD)*RKP - YDR*RNR*(RIX-RKPD)
     1 -RNDR*(-RKRD)*YP + RNDR*RKR*YPD + RKDR*YRD*RNP
     2 + RKDR*(WT-YR)*(-RNPD) -RNDR*YRD*RKP
     3 - (WT-YR)*RNDR*(RIX-RKPD) + RKDR*(RIZ-RNRD)*YP
     4 - RKDR*RNR*YPD + YDR*(-RKRD)*RNP + YDR*RKR*(-RNPD)
C
C
C
```

HORLIN.FORT (cont.)

```
      CBETA = -YDR*(RIZ-RNRD)*RKPHI + YDR*RNR*RKP
     1 -RNDR*(-RKRD)*YPHI +RNDR*RKR*YP + RKDR*YRD*RNPHI
     2 -RKDR*(WT-YR)*RNP - RNDR*YRD*RKPHI + (WT-YR)*RNDR*RKP
     3 +RKDR*(RIZ-RNRD)*YPHI - RNR*RKDR*YP + YDR*(-RKRD)*RNPHI
     4 -YDR*RKR*RNP
C23456789
C
C
      DBETA = YDR*RNR*RKPHI + RNDR*RKR*YPHI - RKDR*(WT-YR)*RNPHI
     1 + (WT-YR)*RNDR*RKPHI -RNR*RKDR*YPHI - YDR*RKR*RNPHI
      IF(J)32,32,31
 31    ABETA = ABETA*XLU4*U
      BBETA = BBETA*XLU3*U
      CBETA = CBETA*XLU2*U
      DBETA = DBETA*XLU*U
 32    WRITE(5,52)
      WRITE(6,52)
 52    FORMAT(1X,'SIDESLIP POLYNOMIAL COEFFS.',/)
      WRITE(5,53)
      WRITE(6,53)
 53    FORMAT(7X,'ABETA',9X,'BBETA',8X,'CBETA',8X,'DBETA')
      WRITE(5,100)ABETA,BBETA,CBETA,DBETA
      WRITE(6,100)ABETA,BBETA,CBETA,DBETA
      NDEG = 3
      A2(1) = ABETA
      A2(2) = BBETA
      A2(3) = CBETA
      A2(4) = DBETA
      CALL ZPOLR(A2,NDEG,Z2,IER)
      WRITE(5,54)
      WRITE(6,54)
 54    FORMAT(1X,'ROOTS')
      WRITE(5,205)
      WRITE(6,205)
      WRITE(5,201)Z2
      WRITE(6,201)Z2
C
C
C
      APHI = RKDR*(WT-YVD)*(RIZ-RNRD) - YDR*RNVD*(-RKRD)
     1 + RNDR*RKVD*YRD - RKDR*RNVD*YRD + YDR*RKVD*(RIZ-RNRD)
     2 -RNDR*(WT-YVD)*(-RKRD)
C
C
C
      BPHI = -RKDR*(WT-YVD)*RNR - RKDR*YV*(RIZ-RNRD)
     1 + YDR*RNVD*RKR - YDR*RNV*(-RKRD) - RNDR*RKVD*(WT-YR)
     2 + RNDR*RKV*YRD + RKDR*RNVD*(WT-YR) - RKDR*RNV*YRD
     3 - YDR*RKVD*RNR + YDR*RKV*(RIZ-RNRD) + RNDR*(WT-YVD)*RKR
     4 + RNDR*YV*(-RKRD)
C
C
C
      CPHI= RKDR*YV*RNR + YDR*RNV*RKR - RNDR*RKV*(WT-YR)
     1 + RKDR*RNV*(WT-YR) - YDR*RKV*RNR -RNDR*YV*RKR
      IF(J)42,42,41
 41    APHI = APHI*XLU3
      BPHI = BPHI*XLU2
      CPHI =CPHI*XLU
C
```

HORLIN.FORT (cont.)

```
C
 42    WRITE(5,105)
      WRITE(6,105)
 105   FORMAT(1X,'ROLL POLYNOMIAL COEFFS.',/)
      WRITE(5,106)
      WRITE(6,106)
 106   FORMAT(7X,'APHI',11X,'BPHI',11X,'CPHI')
      WRITE(5,100)APHI,BPHI,CPHI
      WRITE(6,100)APHI,BPHI,CPHI
C
C
C
      NDEG = 2
      A3(1) = APHI
      A3(2) = BPHI
      A3(3) = CPHI
      CALL ZPOLR(A3,NDEG,Z3,IER)
      WRITE(5,202)
      WRITE(6,202)
 202   FORMAT(1X,'ROOTS')
      WRITE(5,205)
      WRITE(6,205)
      WRITE(5,201)Z3
      WRITE(6,201)Z3
C
C
      APSI = RNDR*(WT-YVD)*(RIX-RKPD) + RKDR*RNVD*YPD
     1 - YDR*RKVD*(-RNPD) + YDR*RNVD*(RIX-RKPD)
     2 - RNDR*RKVD*YPD - RKDR*(WT-YVD)*(-RNPD)
C
C
C
      BPSI = -RNDR*(WT-YVD)*RKP - RNDR*YV*(RIX-RKPD)
     1 + RKDR*RNVD*YP +RKDR*RNV*YPD +YDR*RKVD*RNP
     2 -YDR*RKV*(-RNPD) - YDR*RNVD*RKP +YDR*RNV*(RIX-RKPD)
     3 - RNDR*RKVD*YP - RNDR*RKV*YPD + RKDR*(WT-YVD)*RNP
     4 + RKDR*YV*(-RNPD)
C
C
C
      CPSI = -RNDR*(WT-YVD)*RKPHI + RNDR*YV*RKP +RKDR*RNVD*YPHI
     1 + RKDR*RNV*YP +YDR*RKVD*RNPHI +YDR*RKV*RNP - YDR*RNVD*RKPHI
     2 - YDR*RNV*RKP - RNDR*RKVD*YPHI - RNDR*RKV*YP
     3 + RKDR*(WT-YVD)*RNPHI - RKDR*YV*RNP
C
C
C
      DPSI = RNDR*YV*RKPHI + RKDR*RNV*YPHI + YDR*RKV*RNPHI
     1 - YDR*RNV*RKPHI - RNDR*RKV*YPHI - RKDR*YV*RNPHI
      IF(J)62,62,61
 61    APSI = APSI*XLU3
      BPSI = BPSI*XLU2
      CPSI = CPSI*XLU
 62    WRITE(5,60)
      WRITE(6,60)
 60    FORMAT(1X,'YAW POLYNOMIAL COEFFS.',/)
      WRITE(5,70)
      WRITE(6,70)
 70    FORMAT(7X,'APSI',11X,'BPSI',11X,'CPSI',11X,'DPSI')
      WRITE(5,100)APSI,BPSI,CPSI,DPSI
```

HORLIN.FORT (cont.)

```
WRITE(6,100)APSI,BPSI,CPSI,DPSI
NDEG = 3
A4(1) = APSI
A4(2) = BPSI
A4(3) = CPSI
A4(4) = DPSI
CALL ZPOLR(A4,NDEG,Z4,IER)
WRITE(5,202)
WRITE(6,202)
WRITE(5,205)
WRITE(6,205)
WRITE(5,201)Z4
WRITE(6,201)Z4
STOP
END
```

APPENDIX D

AXIAL FORCE

$$m\left[\dot{u} - vr + wq - x_G (q^2 + r^2) + y_G (pq - \dot{r}) + z_G (pr + \dot{q})\right] =$$

$$+ \frac{\rho}{2} \ell^4 \left[X_{qq}{}' q^2 + X_{rr}{}' r^2 + X_{rp}{}' rp\right]$$

$$+ \frac{\rho}{2} \ell^3 \left[X_{\dot{u}}{}' \dot{u} + X_{vr}{}' vr + X_{wq}{}' wq\right]$$

$$+ \frac{\rho}{2} \ell^2 \left[X_{uu}{}' u^2 + X_{vv}{}' v^2 + X_{ww}{}' w^2\right]$$

$$+ \frac{\rho}{2} \ell^2 u^2 \left[X_{\delta r\, \delta r}{}' \delta r^2 + X_{\delta s\, \delta s}{}' \delta s^2 + X_{\delta b\, \delta b}{}' \delta b^2\right]$$

$$+ \tfrac{1}{2} \rho \ell^2 \left[a_i u^2 + b_i u u_c + c_i u_c{}^2\right]$$

$$- (W - B) \sin \theta$$

$$+ \frac{\rho}{2} \ell^2 \left[X_{vv\eta}{}' v^2 + X_{ww\eta}{}' w^2 + X_{\delta r \delta r \eta}{}' \delta_r{}^2 u^2\right.$$

$$\left. + X_{\delta s \delta s \eta}\, \delta_s{}^2 u^2\right] (\eta - 1)$$

LATERAL FORCE

$$m\left[\dot{v} - wp + ur - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(qp + \dot{r})\right] =$$

$$+\frac{\rho}{2}\ell^4\left[Y_{\dot{r}}'\dot{r} + Y_{\dot{p}}'\dot{p} + Y_{p|p|}'p|p| + Y_{pq}'pq + Y_{qr}'qr\right]$$

$$+\frac{\rho}{2}\ell^3\left[Y_{\dot{v}}'\dot{v} + Y_{vq}'vq + Y_{wp}'wp + Y_{wr}'wr\right]$$

$$+\frac{\rho}{2}\ell^3\left[Y_r'ur + Y_p'up + Y_{|r|\delta r}'u|r|\delta r + Y_{v|r|}'\frac{v}{|v|}\left|(v^2 + w^2)^{\frac{1}{2}}\right||r|\right]$$

$$+\frac{\rho}{2}\ell^2\left[Y_*'u^2 + Y_v'uv + Y_{v|v|}'v\left|(v^2 + w^2)^{\frac{1}{2}}\right|\right]$$

$$+\frac{\rho}{2}\ell^2\left[Y_{vw}'vw + Y_{\delta r}'u^2\delta r\right]$$

$$+(W - B)\cos\theta\sin\phi$$

$$+\frac{\rho}{2}\ell^3 Y_{r\eta}'ur(\eta-1)$$

$$+\frac{\rho}{2}\ell^2\left[Y_{v\eta}'uv + Y_{v|v|\eta}'v\left|(V^2 + w^2)^{\frac{1}{2}}\right| + Y_{\delta r\eta}'\delta_r u^2\right](\eta-1)$$

NORMAL FORCE

$$m\left[\dot{w} - uq + vp - z_G\,(p^2 + q^2) + x_G\,(rp - \dot{q}) + y_G\,(rq + \dot{p})\right]$$

$$+\frac{\rho}{2}\,\ell^4\left[Z_{\dot{q}}'\,\dot{q} + Z_{pp}'\,p^2 + Z_{rr}'\,r^2 + Z_{rp}'\,rp\right]$$

$$+\frac{\rho}{2}\,\ell^3\left[Z_{\dot{w}}'\,\dot{w} + Z_{vr}'\,vr + Z_{vp}'\,vp\right]$$

$$+\frac{\rho}{2}\,\ell^3\left[Z_q'\,uq + Z_{|q|\delta s}'\,u|q|\delta s + Z_{w|q|}'\,\frac{w}{|w|}\left|(v^2 + w^2)^{\frac{1}{2}}\right|\,|q|\right]$$

$$+\frac{\rho}{2}\,\ell^2\left[Z_*'\,u^2 + Z_w'\,uw + Z_{w|w|}'\,w\left|(v^2 + w^2)^{\frac{1}{2}}\right|\right]$$

$$+\frac{\rho}{2}\,\ell^2\left[Z_{|w|}'\,u|w| + Z_{ww}'\,\left|w\,(v^2 + w^2)^{\frac{1}{2}}\right|\right]$$

$$+\frac{\rho}{2}\,\ell^2\left[Z_{vv}'\,v^2 + Z_{\delta s}'\,u^2\,\delta s + Z_{\delta b}'\,u^2\,\delta b\right]$$

$$+ (W - B)\cos\theta\cos\phi$$

$$+\frac{\rho}{2}\,\ell^3\,Z_{q\eta}'\,uq\,(\eta - 1)$$

$$+\frac{\rho}{2}\,\ell^2\left[Z_{w\eta}'\,uw + Z_{w|w|\eta}'\,w\left|(v^2 + w^2)^{\frac{1}{2}}\right| + Z_{\delta s\eta}'\,\delta_s u^2\right](\eta - 1)$$

ROLLING MOMENT

$$I_x \dot{p} + (I_z - I_y)\, qr + (\dot{r} + pq)\, I_{xz} + (r^2 - q^2)\, I_{yz} + (pr - \dot{q})\, I_{xy}$$

$$+ m \left[y_G (\dot{w} - uq + vp) - z_G (\dot{v} - wp + ur) \right] =$$

$$+ \frac{\rho}{2} \ell^5 \left[K_{\dot{p}}{}' \dot{p} + K_{\dot{r}}{}' \dot{r} + K_{qr}{}' qr + K_{pq}{}' pq + K_{p|p|}{}' p|p| \right]$$

$$+ \frac{\rho}{2} \ell^4 \left[K_p{}' p + K_r{}' ur + K_{\dot{v}}{}' \dot{v} \right]$$

$$+ \frac{\rho}{2} \ell^4 \left[K_{vq}{}' vq + K_{wp}{}' wp + K_{wr}{}' wr \right]$$

$$+ \frac{\rho}{2} \ell^3 \left[K_*{}' u^2 + K_v{}' uv + K_{v|v|}{}' v \left| (v^2 + w^2)^{\frac{1}{2}} \right| \right]$$

$$+ \frac{\rho}{2} \ell^3 \left[K_{vw}{}' vw + K_{\delta r}{}' u^2 \delta r \right]$$

$$+ (y_G W - y_B B) \cos\theta \cos\phi - (z_G W - z_B B) \cos\theta \sin\phi \,]$$

$$+ \frac{\rho}{2} \ell^3 K_{*\eta}{}' u^2 (\eta - 1)$$

PITCHING MOMENT

$$I_y \dot{q} + (I_x - I_z)\, rp - (\dot{p} + qr)\, I_{xy} + (p^2 - r^2)\, I_{zx} + (qp - \dot{r})\, I_{yz}$$

$$+ m\left[z_G\,(\dot{u} - vr + wq) - x_G\,(\dot{w} - uq + vp)\right] =$$

$$+\frac{\rho}{2}\,\ell^5\left[M_{\dot{q}}'\,\dot{q} + M_{pp}'\,p^2 + M_{rr}'\,r^2 + M_{rp}'\,rp + M_{q|q|}'\,q|q|\right]$$

$$+\frac{\rho}{2}\,\ell^4\left[M_{\dot{w}}'\,\dot{w} + M_{vr}'\,vr + M_{vp}'\,vp\right]$$

$$+\frac{\rho}{2}\,\ell^4\left[M_q'\,uq + M_{|q|\delta s}'\,u|q|\delta s + M_{|w|q}'\,|(v^2 + w^2)^{\frac{1}{2}}|\,q\right]$$

$$+\frac{\rho}{2}\,\ell^3\left[M_*'\,u^2 + M_w'\,uw + M_{w|w|}'\,w\,|(v^2 + w^2)^{\frac{1}{2}}|\right]$$

$$+\frac{\rho}{2}\,\ell^3\left[M_{|w|}'\,u|w| + M_{ww}'\,|w\,(v^2 + w^2)^{\frac{1}{2}}|\right]$$

$$+\frac{\rho}{2}\,\ell^3\left[M_{vv}'\,v^2 + M_{\delta s}'\,u^2\,\delta s + M_{\delta b}'\,u^2\,\delta b\right]$$

$$-\,(x_G\,W - x_B\,B)\cos\theta\cos\phi - (z_G\,W - z_B\,B)\sin\theta$$

$$+\frac{\rho}{2}\,\ell^4\,M_{q\eta}'\,uq\,(\eta-1)$$

$$+\frac{\rho}{2}\,\ell^3\left[M_{w\eta}'\,uw + M_{w|w|\eta}'\,w\,|(v^2 + w^2)^{\frac{1}{2}}| + M_{\delta s\eta}'\,\delta_s u^2\right](\eta-1)$$

YAWING MOMENT

$$I_z \dot{r} + (I_y - I_x)\, pq - (\dot{q} + rp)\, I_{yz} + (q^2 - p^2)\, I_{xy} + (rq - \dot{p})\, I_{zx}$$

$$+ m\left[x_G\,(\dot{v} - wp + ur) - y_G\,(\dot{u} - vr + wq)\right] =$$

$$+\frac{\rho}{2}\,\ell^5\left[N_{\dot{r}}'\,\dot{r} + N_{\dot{p}}'\,\dot{p} + N_{pq}'\,pq + N_{qr}'\,qr + N_{r|r|}'\,r|r|\right]$$

$$+\frac{\rho}{2}\,\ell^4\left[N_{\dot{v}}'\,\dot{v} + N_{wr}'\,wr + N_{wp}'\,wp + N_{vq}'\,vq\right]$$

$$+\frac{\rho}{2}\,\ell^4\left[N_p'\,up + N_r'\,ur + N_{|r|\delta r}'\,u|r|\delta r + N_{|v|r}'\,\left|(v^2 + w^2)^{\frac{1}{2}}\right| r\right]$$

$$+\frac{\rho}{2}\,\ell^3\left[N_*'\,u^2 + N_v'\,uv + N_{v|v|}'\,v\,\left|(v^2 + w^2)^{\frac{1}{2}}\right|\right]$$

$$+\frac{\rho}{2}\,\ell^3\left[N_{vw}'\,vw + N_{\delta r}'\,u^2\,\delta r\right]$$

$$+ (x_G\,W - x_B\,B)\cos\theta\,\sin\phi + (y_G\,W - y_B\,B)\sin\theta$$

$$+\frac{\rho}{2}\,\ell^4\,N_{r\eta}'\,ur\,(\eta - 1)$$

$$+\frac{\rho}{2}\,\ell^3\left[N_{v\eta}'\,uv + N_{v|v|\eta}'\,v\left|(v^2 + w^2)^{\frac{1}{2}}\right| + N_{\delta r\eta}'\,\delta_r\,u^2\right](\eta - 1)$$

KINEMATIC RELATIONS

$$U^2 = u^2 + v^2 + w^2$$

$$\dot{z}_o = -u \sin \theta + v \cos \theta \sin \phi + w \cos \theta \cos \phi$$

$$\dot{\phi} = p + \dot{\psi} \sin \theta$$

$$\dot{\theta} = \frac{q - \dot{\psi} \cos \theta \sin \phi}{\cos \phi}$$

$$\dot{\psi} = \frac{r + \theta \sin \phi}{\cos \theta \cos \phi}$$

APPENDIX E

```
C       ******************************************************************
C       THIS SUBROUTINE CALCULATES STATE DERIVATIVES OF THE TRUTH
C       MODEL (6-DOF NONLINEAR MODEL) FOR THE FOLLOWING EQS.
C
C                  XDOT = F(X,U) + W
C
        SUBROUTINE TRUMOD
        IMPLICIT REAL*8(A-H,O-Z)
C23456789
        COMMON /MASS/ BM,BL,BF,XG,YG,ZG,XB,YB,ZB,BIX,BIY,BIZ,
     1            BIXY,BIYZ,BIZX
C
        COMMON /XCOEFF/ AI,BI,CI,XDRDR,XDRDRE,XDSDS,XDSDSE,XQQ,XRP,XRR,
     1             XUDOT,XUU,XVR,XVV,XVVE,XWQ,XWW,XWWE
C
        COMMON /YCOEFF/ YDR,YDRE,YP,YPDOT,YPP,YPQ,YQR,YR,YRDOT,YRDR,
     1             YRE,YS,YV,YVDOT,YVE,YVQ,YVR,YVV,YVVE,YVW,
     1             YWP,YWR
C
        COMMON /ZCOEFF/ ZAW,ZDS,ZDSE,ZPP,ZQ,ZQDOT,ZQDS,ZQE,ZRP,ZRR,ZS,
     1             ZVP,ZVR,ZVV,ZW,ZWAW,ZWDOT,ZWE,ZWQ,ZWW,ZWWE
C
        COMMON /KCOEFF/ HKDR,HKP,HKPDOT,HKPP,HKPQ,HKQR,HKR,HKRDOT,
     1             HKS,HKSE,HKV,HKVDOT,HKVQ,HKVV,HKVW,HKWP,HKWR
C
        COMMON /MCOEFF/ HMAW,HMDS,HMDSE,HMPP,HMQ,HMQDOT,HMQDS,HMQE,
     1             HMQQ,HMRP,HMRR,HMS,HMVP,HMVR,HMVV,HMW,HMWAW,
     1             HMWDOT,HMWE,HMWQ,HMWW,HMWWE
C
        COMMON /NCOEFF/ HNDR,HNDRE,HNP,HNPDOT,HNPQ,HNQR,HNR,HNRDOT,
     1             HNRDR,HNRE,HNRR,HNS,HNV,HNVDOT,HNVE,HNVQ,HNVR,
     1             HNVV,HNVVE,HNVW,HNWP,HNWR
C
        COMMON /TRUMOD/ TX(12),TXDOT(12),TXSIG(12),TXOM(12),TXOSIG(12),
     1             TMINV(6,6)'
C23456789
        COMMON /INPUT/ UCON(3)
C
        COMMON /SYSTEM/ TIME,DELT,FIRST
C
        COMMON /FLUID/ VFE(3)
```

```
      LOGICAL FIRST
C
      DIMENSION TM(6,6),FM(6),XMDOT(6),DUMV1(3),DUMV2(3),CBE(3,3),
     1          W(12),DUM(6,6)
C23456789
C     PHYSICAL CONSTANTS
C
      DATA RHO/1.9905/,GR/32.2/
C
C     INITIALIZE CONSTANTS.
C
      R2L2=RHO/2. * BL*BL
      R2L3=R2L2*BL
      R2L4=R2L3*BL
      R2L5=R2L4*BL
      WW=BM*GR - BF
      VEL=SQRT(TX(1)**2+TX(2)**2+TX(3)**2)
      ETA=UCON(3)/VEL
C
C     COMPUTE INERTIAL COEFFICIENT MATRIX.  THIS MATRIX IS COMPUTED
C     ONLY ONCE AT THE BEGINNING OF EXECUTION.
C
      IF (.NOT. FIRST) GO TO 10
C
      DO 5 I=1,6
      DO 5 J=1,6
    5 TM(I,J)=0.0
C23456789
      TM(1,1) = BM - R2L3*XUDOT
      TM(1,5) = BM*ZG
      TM(1,6) = -BM*YG
      TM(2,2) = BM - R2L3*YVDOT
      TM(2,4) = -BM*ZG - R2L4*YPDOT
      TM(2,6) = BM*XG - R2L4*YRDOT
      TM(3,3) = BM - R2L3*ZWDOT
      TM(3,4) = BM*YG
      TM(3,5) = -BM*XG - R2L4*ZQDOT
      TM(4,2) = -BM*ZG - R2L4*HKVDOT
      TM(4,3) = BM*YG
      TM(4,4) = BIX - R2L5*HKPDOT
      TM(4,5) = -BIXY
      TM(4,6) = -BIZX - R2L5*HKRDOT
```

```
      TM(5,1) = BM*ZG
      TM(5,3) = -BM*XG - R2L4*HMWDOT
      TM(5,4) = -BIXY
      TM(5,5) = BIY - R2L5*HMQDOT
      TM(5,6) = -BIYZ
      TM(6,1) = -BM*YG
      TM(6,2) = BM*XG - R2L4*HNVDOT
      TM(6,4) = -BIZX - R2L5*HNPDOT
      TM(6,5) = -BIYZ
      TM(6,6) = BIZ - R2L5*HNRDOT
C
C     INVERSE TM
C
      CALL MATINV(TM,TMINV,6,DUM,IER)
C
C     IF IER=1, TM IS A SINGULAR MATRIX.  *******************
C
   10 CONTINUE
C
C     COMPUTE AXIAL FORCE
C
      X1 = BM*(TX(2)*TX(6)-TX(3)*TX(5))
      X2 = BM*XG*(TX(5)*TX(5)+TX(6)*TX(6))
      X3 = BM*YG*TX(4)*TX(5)
      X4 = BM*ZG*TX(4)*TX(6)
      X5 = R2L4*(XQQ*TX(5)*TX(5)+XRR*TX(6)*TX(6)+XRP*TX(6)*TX(4))
      X6 = R2L3*(XVR*TX(2)*TX(6)+XWQ*TX(3)*TX(5))
      X7 = R2L2*(XUU*TX(1)*TX(1)+XVV*TX(2)*TX(2)+XWW*TX(3)*TX(3))
      X8 = R2L2*TX(1)*TX(1)*(XDRDR*UCON(1)*UCON(1)
     1                      +XDSDS*UCON(2)*UCON(2))
      X9 = R2L2*(AI*TX(1)*TX(1)+BI*TX(1)*UCON(3)+CI*UCON(3)**2)
      X10 = WW*SIN(TX(11))
      X11 = R2L2*(XVVE*TX(2)**2+XWWE*TX(3)**2+XDRDRE*UCON(1)**2
     1         *TX(1)**2+XDSDSE*UCON(2)**2*TX(1)**2)*(ETA-1.)
C23456789
      FM(1) = X1+X2-X3-X4+X5+X6+X7+X8+X9-X10+X11
C
C     COMPUTE LATERAL FORCE
C
      Y1 = BM*(TX(3)*TX(4)-TX(1)*TX(6))
      Y2 = BM*YG*(TX(6)**2+TX(4)**2)
      Y3 = BM*ZG*TX(5)*TX(6)
```

```
      Y4 = BM*XG*TX(5)*TX(4)
      Y5 = R2L4*(YPP*TX(4)*ABS(TX(4))+YPQ*TX(4)*TX(5)
     1       +YQR*TX(5)*TX(6))
      Y6 = R2L3*(YVQ*TX(2)*TX(5)+YWP*TX(3)*TX(4)+YWR*TX(3)*TX(6))
      Y7 = R2L3*(YR*TX(1)*TX(6)+YP*TX(1)*TX(4)
     1       +YRDR*TX(1)*ABS(TX(6))*UCON(1)
     1       +YVR*TX(2)/ABS(TX(2))*SQRT(TX(2)**2+TX(3)**2)
     1             *ABS(TX(6)))
      Y8 = R2L2*(YS*TX(1)**2+YV*TX(1)*TX(2)
     1       +YVV*TX(2)*SQRT(TX(2)**2+TX(3)**2))
      Y9 = R2L2*(YVW*TX(2)*TX(3)+YDR*TX(1)**2*UCON(1))
      Y10 = WW*COS(TX(11))*SIN(TX(10))
      Y11 = R2L3*YRE*TX(1)*TX(6)*(ETA-1.)
      Y12 = R2L2*(YVE*TX(1)*TX(2)+YVVE*TX(2)*SQRT(TX(2)**2+TX(3)**2)
     1        +YDRE*UCON(1)*TX(1)**2)*(ETA-1.)
C23456789
      FM(2) = Y1+Y2-Y3-Y4+Y5+Y6+Y7+Y8+Y9+Y10+Y11+Y12
C
C     COMPUTE NORMAL FORCE
C
      Z1 = BM*(TX(1)*TX(5)-TX(2)*TX(4))
      Z2 = BM*ZG*(TX(4)**2+TX(5)**2)
      Z3 = BM*XG*TX(6)*TX(4)
      Z4 = BM*YG*TX(6)*TX(5)
      Z5 = R2L4*(ZPP*TX(4)**2+ZRR*TX(6)**2+ZRP*TX(6)*TX(4))
      Z6 = R2L3*(ZVR*TX(2)*TX(6)+ZVP*TX(2)*TX(4))
      DDUM = TX(3)*SQRT(TX(2)**2+TX(3)**2)
      Z7 = R2L3*(ZQ*TX(1)*TX(5)+ZQDS*TX(1)*ABS(TX(5))*UCON(2)
     1       +ZWQ*DDUM/ABS(TX(3))*ABS(TX(5)))
      Z8 = R2L2*(ZS*TX(1)**2+ZW*TX(1)*TX(3)+ZWAW*DDUM)
      Z9 = R2L2*(ZAW*TX(1)*ABS(TX(3))+ZWW*ABS(DDUM))
      Z10 = R2L2*(ZVV*TX(2)**2+ZDS*TX(1)**2*UCON(2))
      Z11 = WW*COS(TX(11))*COS(TX(10))
      Z12 = R2L3*ZQE*TX(1)*TX(5)*(ETA-1.)
      Z13 = R2L2*(ZWE*TX(1)*TX(3)+ZWWE*DDUM+ZDSE*UCON(2)*TX(1)**2)
     1        *(ETA-1.)
C
      FM(3) = Z1+Z2-Z3-Z4+Z5+Z6+Z7+Z8+Z9+Z10+Z11+Z12+Z13
C
C     COMPUTE ROLLING MOMENT
C
      HK1 = (BIZ-BIY)*TX(5)*TX(6)
```

```
      HK2 = BIZX*TX(4)*TX(4)
      HK3 = BIYZ*(TX(6)**2-TX(5)**2)
      HK4 = BIXY*TX(4)*TX(6)
      HK5 = BM*YG*(TX(1)*TX(5)-TX(2)*TX(4))
      HK6 = BM*ZG*(-TX(3)*TX(4)+TX(1)*TX(6))
      HK7 = R2L5*(HKQR*TX(5)*TX(6)+HKPQ*TX(4)*TX(5)
     1        +HKPP*TX(4)*ABS(TX(4)))
      HK8 = R2L4*(HKP*TX(1)*TX(4)+HKR*TX(1)*TX(6))
      HK9 = R2L4*(HKVQ*TX(2)*TX(5)+HKWP*TX(3)*TX(4)+HKWR*TX(3)*TX(6))
      HK10 = R2L3*(HKS*TX(1)**2+HKV*TX(1)*TX(2)+HKVV*TX(2)
     1        *SQRT(TX(2)**2+TX(3)**2))
      HK11 = R2L3*(HKVW*TX(2)*TX(3)+HKDR*TX(1)**2*UCON(1))
      HK12 = (YG*BM*GR-YB*BF)*COS(TX(11))*COS(TX(10))
     1    -(ZG*BM*GR-ZB*BF)*COS(TX(11))*SIN(TX(10))
      HK13 = R2L3*HKSE*TX(1)**2*(ETA-1.)
C23456789
      FM(4) = -HK1+HK2-HK3-HK4+HK5+HK6+HK7+HK8+HK9+HK10+HK11+HK12
     1    +HK13
C
C     COMPUTE PITCHING MOMENT
C
      HM1 = (BIX-BIZ)*TX(6)*TX(4)
      HM2 = BIXY*TX(5)*TX(6)
      HM3 = BIZX*(TX(4)**2-TX(6)**2)
      HM4 = BIYZ*TX(5)*TX(4)
      HM5 = BM*ZG*(TX(2)*TX(6)-TX(3)*TX(5))
      HM6 = BM*XG*(-TX(1)*TX(5)+TX(2)*TX(4))
      HM7 = R2L5*(HMPP*TX(4)**2+HMRR*TX(6)**2+HMRP*TX(6)*TX(4)
     1        +HMQQ*TX(5)*ABS(TX(5)))
      HM8 = R2L4*(HMVR*TX(2)*TX(6)+HMVP*TX(2)*TX(4))
      HM9 = R2L4*(HMQ*TX(1)*TX(5)+HMQDS*TX(1)*ABS(TX(5))*UCON(2)
     1        +HMWQ*DDUM/TX(3)*TX(5))
      HM10 = R2L3*(HMS*TX(1)**2+HMW*TX(1)*TX(3)+HMWAW*DDUM)
      HM11 = R2L3*(HMAW*TX(1)*ABS(TX(3))+HMWW*ABS(DDUM))
      HM12 = R2L3*(HMVV*TX(2)**2+HMDS*TX(1)**2*UCON(2))
      HM13 = (XG*BM*GR-XB*BF)*COS(TX(11))*COS(TX(10))
     1    +(ZG*BM*GR-ZB*BF)*SIN(TX(11))
      HM14 = R2L4*HMQE*TX(1)*TX(5)*(ETA-1.)
      HM15 = R2L3*(HMWE*TX(1)*TX(3)+HMWWE*DDUM
     1         +HMDSE*UCON(2)*TX(1)**2)*(ETA-1.)
C
      FM(5) = -HM1+HM2-HM3-HM4+HM5+HM6+HM7+HM8+HM9+HM10+HM11+HM12
```

```
     1         -HM13+HM14+HM15
C
C        COMPUTE YAWING MOMENT
C
         HN1 = (BIY-BIX)*TX(4)*TX(5)
         HN2 = BIYZ*TX(6)*TX(4)
         HN3 = BIXY*(TX(5)**2-TX(4)**2)
         HN4 = BIZX*TX(6)*TX(5)
         HN5 = BM*XG*(-TX(3)*TX(4)+TX(1)*TX(6))
         HN6 = BM*YG*(-TX(2)*TX(6)+TX(3)*TX(5))
         HN7 = R2L5*(HNPQ*TX(4)*TX(5)+HNQR*TX(5)*TX(6)+HNRR*TX(6)
     1           *ABS(TX(6)))
         HN8 = R2L4*(HNWR*TX(3)*TX(6)+HNWP*TX(3)*TX(4)
     1           +HNVQ*TX(2)*TX(5))
         DDUM = TX(2)*SQRT(TX(2)**2+TX(3)**2)
         HN9 = R2L4*(HNP*TX(1)*TX(4)+HNR*TX(1)*TX(6)+HNRDR*TX(1)
     1           *ABS(TX(6))*UCON(1)+HNVR*DDUM/TX(2)*TX(6))
         HN10 = R2L3*(HNS*TX(1)**2+HNV*TX(1)*TX(2)+HNVV*DDUM)
         HN11 = R2L3*(HNVW*TX(2)*TX(3)+HNDR*TX(1)**2*UCON(1))
         HN12 = (XG*BM*GR-XB*BF)*COS(TX(11))*SIN(TX(10))
     1      +(YG*BM*GR-YB*BF)*SIN(TX(11))
         HN13 = R2L4*HNRE*TX(1)*TX(6)*(ETA-1.)
         HN14 = R2L3*(HNVE*TX(1)*TX(2)+HNVVE*DDUM+HNDRE*UCON(1)
     1            *TX(1)**2)*(ETA-1.)
C23456789
         FM(6) = -HN1+HN2-HN3-HN4-HN5+HN6+HN7+HN8+HN9+HN10+HN11+HN12
     1      +HN13+HN14
C
C        MULTIPLY TMINV AND FM TO OBTAIN XMDOT WHICH IS EQUIVALENT TO
C        TXDOT(1) THROUGH TXDOT(6).
C
         CALL MULT(TMINV,FM,XMDOT,6,6,1)
C
         DO 20 I=1,6
   20    TXDOT(I)=XMDOT(I)
C
C        COMPUTE TXDOT(7) THROUGH TXDOT(9) USING
C
C            [TXDOT(7),TXDOT(8),TXDOT(9)]=CBE*[TX(1),TX(2),
C               TX(3)]+VFE
C
C        WHERE  CBE=BODY TO EARTH FRAME TRANSFORMATION MATRIX
```

```
C               VFE=FLUID VELOCITY IN E-FRAME
C
      DO 30 I=1,3
      DUMV1(I)=TX(I)
      DUMV2(I)=TX(I+9)
   30 CONTINUE
C
      CALL TMTRIX(DUMV2,CBE)
      CALL MULT(CBE,DUMV1,DUMV2,3,3,1)
      CALL ADD(DUMV2,VFE,DUMV1,3,1)
C
      DO 40 I=1,3
   40 TXDOT(I+6)=DUMV1(I)
C
C     COMPUTE TXDOT(10) THROUGH TXDOT(12)
C
      SP=DSIN(TX(10))
      TT=DTAN(TX(11))
      CP=DCOS(TX(10))
      CT=DCOS(TX(11))
C
      TXDOT(10) = TX(4) + SP*TT*TX(5) +CP*TT*TX(6)
      TXDOT(11) = CP*TX(5) - SP*TX(6)
      TXDOT(12) = SP/CT*TX(5) + CP/CT*TX(6)
C23456789
C     ADD NOISE TERM
C
      DO 50 I=1,12
      TXDOT(I)=TXDOT(I) + GAUSS(TXSIG(I))
   50 CONTINUE
C
      RETURN
      END
```

```
C       BLOCK DATA DEFINES ALL THE COMMON BLOCKS AND VARIABLES,
C       AND INITIATES ALL CONSTANTS.
        BLOCK DATA
C
        IMPLICIT REAL*8(A-H,O-Z)
C
C       COMMON /SYSTEM/ CONTAINS SIMULATION PARAMETERS
C
        COMMON /SYSTEM/ TIME,DELT,FIRST
C
C       TIME = SIMULATION TIME
C       DELT = SIMULATION TIME INCREMENT
C       FIRST = FLAG TO INDICATE TIME=0.0
C23456789
C       COMMON /MASS/ CONTAINS MASS PROPERTIES.
        COMMON /MASS/ BM,BL,BF,XG,YG,ZG,XB,YB,ZB,BIX,BIY,BIZ,
     1             BIXY,BIYZ,BIZX
C234567 BM = BODY MASS
C       BL = BODY LENGTH
C       BF = BUOYANCY FORCE, POSITIVE UPWARD
C       XG = X COORDINATE OF CG
C       YG = Y COORDINATE OF CG
C       ZG = Z COORDINATE OF CG
C       XB = X COORDINATE OF CB
C       YB = Y COORDINATE OF CB
C       ZB = Z COORDINATE OF CB
C       BIX = MOMENT OF INERTIA ABOUT X-AXIS
C       BIY = MOMENT OF INERTIA ABOUT Y-AXIS
C       BIZ = MOMENT OF INERTIA ABOUT Z-AXIS
C       BIXY = CROSS PRODUCT OF INERTIA
C       BIYZ = CROSS PRODUCT OF INERTIA
C       BIZX = CROSS PRODUCT OF INERTIA
C234567
        DATA BM/28.7  /,BL/11.5/,BF/923.6 /,
     1    XG/0.     /,YG/ 0.   '/,ZG/ 0.    /,
     1    XB/0.     /,YB/ 0.    /,ZB/ .042/,
     1    BIX/0.     /,BIY/3888 /,BIZ/3888 /,
     1    BIXY/ 0.    /,BIYZ/0.     /,BIZX/ 0.    /
C
C       COMMON /XCOEFF/ CONTAINS AXIAL HYDRODYNAMIC FORCE
C       COEFFICIENTS.
C
```

```
      COMMON /XCOEFF/ AI,BI,CI,XDRDR,XDRDRE,XDSDS,XDSDSE,XQQ,XRP,XRR,
     1              XUDOT,XUU,XVR,XVV,XVVE,XWQ,XWW,XWWE
C234567
      DATA AI/0.    /,BI/ 0./,CI/0./,XDRDR/0./,
     1  XDRDRE/ 0.   /,XDSDS/0./,XDSDSE/0./,XQQ/0.    /,
     1  XRP/0./,XRR/0.    /,XUDOT/.00225/,XUU/0.    /,
     1  XVR/0.    /,XVV/0./,XVVE/0./,XWQ/0./,
     1  XWW/0.    /,XWWE/ 0.   /
C
C     COMMON /YCOEFF/ CONTAINS LATERAL HYDRODYNAMIC FORCE COEFFICIENT.
C23456789
      COMMON /YCOEFF/ YDR,YDRE,YP,YPDOT,YPP,YPQ,YQR,YR,YRDOT,YRDR,
     1              YRE,YS,YV,YVDOT,YVE,YVQ,YVR,YVV,YVVE,YVW,
     1              YWP,YWR
C23456789
      DATA YDR/ 0./,YDRE/0./,YP/ 0.   /,YPDOT/0./,YPP/0.    /,
     1  YPQ/0.    /,YQR/0./,YR/.020415/,YRDOT/.006/,YRDR/0.    /,
     1  YRE/ 0.   /,YS/0./,YV/-.04969/,YVDOT/-.0444/,YVE/0.    /,
     1  YVQ/0./,YVR/0.    /,YVV/0./,YVVE/0.    /,YVW/0.    /,
     1  YWP/ 0.   /,YWR/0.    /
C
C     COMMON /ZCOEFF/ CONTAINS NORMAL HYDRODYNAMIC FORCE COEFFICIENT.
C23456789
      COMMON /ZCOEFF/ ZAW,ZDS,ZDSE,ZPP,ZQ,ZQDOT,ZQDS,ZQE,ZRP,ZRR,ZS,
     1              ZVP,ZVR,ZVV,ZW,ZWAW,ZWDOT,ZWE,ZWQ,ZWW,ZWWE
C
      DATA ZAW/ 0./,ZDS/0./,ZDSE/0./,ZPP/0.    /,ZQ/-.018066/,
     1  ZQDOT/-.006/,ZQDS/0./,ZQE/0.    /,ZRP/0.    /,
     1  ZRR/0.    /,ZS/0.    /,ZVP/0.    /,ZVR/0./,ZVV/0.    /,
     1  ZW/-.04969/,ZWAW/0./,ZWDOT/-.0444/,ZWE/0.    /,
     1  ZWQ/0.    /,ZWW/0.    /,ZWWE/ 0.   /
C
C     COMMON /KCOEFF/ CONTAINS ROLLING MOMENT COEFFICIENTS.
C
      COMMON /KCOEFF/ HKDR,HKP,HKPDOT,HKPP,HKPQ,HKQR,HKR,HKRDOT,
     1              HKS,HKSE,HKV,HKVDOT,HKVQ,HKVV,HKVW,HKWP,HKWR
C234567891123456789212345678931234567894123456789512345678961234567897I2*********
      DATA HKDR/0.    /,HKP/-.000093/,HKPDOT/-.000037/,HKPP/0./,
     1  HKPQ/0./,HKQR/0.    /,HKR/ 0.   /,HKRDOT/0.    /,
     1  HKS/ 0.   /,HKSE/0.    /,HKV/0.    /,HKVDOT/0.    /,
     1  HKVQ/0.    /,HKVV/ 0.   /,HKVW/ 0.   /,HKWP/  0.   /,
     1  HKWR/0.    /
```

```
C
C       COMMON /MCOEFF/ CONTAINS PITCHING MOMENT COEFFICIENTS.
C
        COMMON /MCOEFF/ HMAW,HMDS,HMDSE,HMPP,HMQ,HMQDOT,HMQDS,HMQE,
     1                HMQQ,HMRP,HMRR,HMS,HMVP,HMVR,HMVV,HMW,HMWAW,
     1                HMWDOT,HMWE,HMWQ,HMWW,HMWWE
C
        DATA HMAW/0.      /,HMDS/0.      /,HMDSE/0.      /,HMPP/ 0.     /,
     1    HMQ/-.014299/,HMQDOT/.005439/,HMQDS/ 0.     /,HMQE/0.      /,
     1    HMQQ/ 0.     /,HMRP/0.      /,HMRR/0.      /,HMS/ 0.     /,
     1    HMVP/0.      /,HMVR/0.      /,HMVV/0.      /,HMW/.012174/,
     1    HMWAW/  0.    /,HMWDOT/-.006/,HMWE/0.      /,HMWQ/0.      /,
     1    HMWW/0.      /,HMWWE/0.      /
C
C       COMMON /NCOEFF/ CONTAINS YAWING MOMENT COEFFICIENTS.
C
        COMMON /NCOEFF/ HNDR,HNDRE,HNP,HNPDOT,HNPQ,HNQR,HNR,HNRDOT,
     1                HNRDR,HNRE,HNRR,HNS,HNV,HNVDOT,HNVE,HNVQ,HNVR,
     1                HNVV,HNVVE,HNVW,HNWP,HNWR
C
        DATA HNDR/0.      /,HNDRE/0.      /,HNP/  0.    /,HNPDOT/ 0. /,
     1    HNPQ/  0.    /,HNQR/ 0.     /,HNR/-.014299/,HNRDOT/.005439/,
     1    HNRDR/ 0.     /,HNRE/ 0.     /,HNRR/0.      /,HNS/0.      /,
     1    HNV/-.00944/,HNVDOT/.006/,HNVE/0.      /,HNVQ/0.      /,
     1    HNVR/ 0.     /,HNVV/  0.    /,HNVVE/ 0.     /,HNVW/0.      /,
     1    HNWP/ 0.     /,HNWR/0.      /
C23456789
C       COMMON /TRUMOD/ CONTAINS VARIABLES OF TRUTH MODEL.
C2345678911234567892123456789312345678941234567895123456789612345678971 2********
        COMMON /TRUMOD/ TX(12),TXDOT(12),TXSIG(12),TXOM(12),TXOSIG(12),
     1                TMINV(6,6),TZSFE(12),TZBIAS(12)
C
C       TX = STATES
C       TXDOT = DERIVATIVES OF STATES
C       TXSIG = PROCESS NOISE STANDARD DEVIATION
C       TXOM = INITIAL STATE MEAN VALUES
C       TXOSIG = INITIAL STATE STANDARD DEVIATION
C       TMINV = INVERSE OF INERTIAL COEFFICIENT MATRIX
C       TZSFE = MEASUREMENT SCALE FACTOR ERROR
C       TZBIAS = MEASUREMENT BIAS ERROR
C
        DATA TX/0.1,0.0001,0.000001,0.,2*0.000001,0.,0.,0.,0.,0.,0./,
```

```
     1    TXDOT/12*0./,TXSIG/12*10./,TXOM/12*0./,
     1    TXOSIG/12*0./,TMINV/36*0./,TZSFE/12*0./,TZBIAS/12*0./
C23456789
C       COMMON /INPUT/ CONTAINS CONTROL INPUT VARIABLES.
C
        COMMON /INPUT/ UCON(3)
C
C       UCON(1) = DELTAR = RUDDER DEFLECTION
C       UCON(2) = DELTAS =        DEFLECTION
C       UCON(3) = UC = COMMANDED FORWARD SPEED
C
        DATA UCON/0.0,0.0,8.445/
C
C       COMMON /HCOEFF/ CONTAINS COEFFICIENTS FOR HORIZONTAL FILTER
C       MODEL.
C
        COMMON /HCOEFF/ FYDOT,FYPDOT,FYVDOT,FYR,FYP,FYV,FYDR,
     1              FKPDOT,FKRDOT,FKP,FKR,FKVDOT,FKV,FKDR,
     1              FNRDOT,FNPDOT,FNVDOT,FNP,FNR,FNV,FNDR
C23456789
        DATA FYDOT/0.     /,FYPDOT/0.     /,FYVDOT/0./,FYR/0./,
     1    FYP/  0.   /,FYV/  0.   /,FYDR/0.     /,FKPDOT/0.     /,
     1    FKRDOT/0.     /,FKP/ 0.    /,FKR/0.     /,FKVDOT/0.     /,
     1    FKV/ 0.    /,FKDR/ 0.    /,FNRDOT/0.     /,FNPDOT/  0.   /,
     1    FNVDOT/0.     /,FNP/ 0.    /,FNR/0.     /,FNV/ 0.    /,
     1    FNDR/0.     /
C
C       COMMON /VCOEFF/ CONTAINS COEFFICIENTS FOR VERTICAL FILTER
C       MODEL.
C
        COMMON /VCOEFF/ FXUDOT,FXU,FZQDOT,FZWDOT,FZQ,FZW,FZDS,
     1              FMQDOT,FMWDOT,FMQ,FMW,FMDS
C
        DATA FXUDOT/ 0.    /,FXU/0.     /,FZQDOT/0./,FZWDOT/0.      /,
     1    FZQ/  0.   /,FZW/ 0.    /,FZDS/0.      /,FMQDOT/ 0.    /,
     1    FMWDOT/0.     /,FMQ/ 0.    /,FMW/ 0.    /,FMDS/0.     /
C23456789
C       COMMON /HMOD/ CONTAINS VARIABLES OF HORIZONTAL FILTER
C       MODEL.
C
        COMMON /HMOD/ XHHAT(5),XHDOT(5),XHSIG(5),XHOM(5),XHOSIG(5),
     1             HMINV(5,5),AH(5,5),BH(5),PH(5,5),QH(5,5),
```

```
     1              RHSIG(5),RH(5,5),HH(5,5)
C
C        XHHAT = HORIZONTAL STATE ESTIMATE
C        XHSIG = MEAN SQUARE-ROOT ERROR OF PROCESS NOISE FOR THE
C                HORIZONTAL FILTER MODEL
C        XHOM = INITIAL GUESS OF HORIZONTAL STATE
C        XHOSIG = MEAN SQUARE-ROOT ERROR IN KNOWLEDGE OF THE INITIAL
C                 STATES
C        HMINV = INVERSE OF INERTIAL COEFFICIENT MATRIX
C        AH,BH,HH = MATRICES IN THE FILTER MODEL
C        PH = COVARIANCE MATRIX
C        QH = PROCESS NOISE ERROR VARIANCE MATRIX
C        RHSIG = MEAN SQUARE-ROOT MEASUREMENT ERROR
C        RH = MEASUREMENT NOISE ERROR VARIANCE MATRIX
C23456789
         DATA XHHAT/5*0./,XHDOT/5*0./,XHSIG/5*0./,XHOM/5*0./,
     1     XHOSIG/5*0./,HMINV/25*0./,AH/25*0./,BH/5*0./,
     1     PH/25*0./,QH/25*0./,RHSIG/5*0./,RH/25*0./,HH/25*0./
C
C        COMMON /VMOD/ CONTAINS VARIABLES OF VERTICAL FILTER MODEL.
C
         COMMON /VMOD/ XVHAT(4),XVDOT(4),XVSIG(4),XVOM(4),XVOSIG(4),
     1              VMINV(4,4),AV(4,4),BV(4),PV(4,4),QV(4,4),
     1              RVSIG(4),RV(4,4),HV(4,4)
C
         DATA XVHAT/4*0./,XVDOT/4*0./,XVSIG/4*0./,XVOM/4*0./,
     1     XVOSIG/4*0./,VMINV/16*0./,AV/16*0./,BV/4*0./,
     1     PV/16*0./,QV/16*0./,RVSIG/4*0./,RV/16*0./,HV/16*0./
C
C        COMMON /MEAS/ CONTAINS MEASUREMENTS
C
         COMMON /MEAS/ ZT(12),ZH(5),ZV(4)
C
C        ZT = TRUTH MODEL MEASUREMENTS
C        ZH = HORIZONTAL MODEL MEASUREMENTS
C        ZV = VERTICAL MODEL MEASUREMENTS
C
         END
```

```
C***************************************************************
      SUBROUTINE MULT ( A, B, C, N, L, M )
        IMPLICIT REAL*8(A-H,O-Z)
C
C**********************************************************************
C
C     PROGRAM NAME - AERODYNAMIC COEFFICIENT ESTIMATION (ACES)
C
C     ROUTINE NAME - MULT                          PROG. NO. XXX
C
C**********************************************************************
C
C     SUBROUTINE MULT ( A, B, C, N, L, M )
C
C     PURPOSE
C        COMPUTE THE MATRIX PRODUCT C = A * B, WITH
C        MULTIPLICATION BY THE ZERO VALUES IN A ELIMINATED.
C
C     ARGUMENTS                                          INPUT/OUTPUT
C        N      - NUMBER OF ROWS IN A AND C                 INPUT
C        L      - NUMBER OF COLUMNS IN A AND ROWS IN B      INPUT
C        M      - NUMBER OF COLUMNS IN B AND C              INPUT
C        A      - NXL MATRIX MULTIPLICAND                   INPUT
C        B      - LXM MATRIX MULTIPLIER                     INPUT
C        C      - NXM MATRIX PRODUCT A*B                    OUTPUT
C
C**********************************************************************
C
      DIMENSION A(1),B(1),C(1)
C23456789
      IF( (N .EQ. 0) .OR. (M .EQ. 0) .OR. (L .EQ. 0) ) RETURN
C
      DO 50 I = 1,N
         IJ = I
         DO 10 J = 1,M
            C(IJ) = 0.0
            IJ = IJ + N
   10    CONTINUE
         IJ = I
         DO 40 J = 1,L
            IF (A(IJ) .EQ. 0.0) GO TO 30
            JKN = I
```

```
         JKL = J
         DO 20 K = 1,M
            C(JKN) = C(JKN) + A(IJ)*B(JKL)
            JKN = JKN + N
            JKL = JKL + L
   20    CONTINUE
   30    IJ = IJ + N
   40    CONTINUE
   50 CONTINUE
C23456789
      RETURN
      END
```

```
C23456789
      SUBROUTINE ADD (A, B, C, N, M )
       IMPLICIT REAL*8(A-H,O-Z)
C
C
C***********************************************************************
C
C     PROGRAM NAME - AERODYNAMIC COEFFICIENT ESTIMATION (ACES)
C
C     ROUTINE NAME - ADD                          PROG. NO. XXX
C
C***********************************************************************
C23456789
C     SUBROUTINE ADD ( A, B, C, N, M )
C
C     PURPOSE
C        COMPUTE C = A + B, WHERE A, B, AND C ARE NXM MATRICES.
C        (NOTE:  A, B, AND C NEED NOT BE DISTINCT.)
C
C     ARGUMENTS                                            INPUT/OUTPUT
C        A      - NXM MATRIX ADDEND                          INPUT
C        B      - NXM MATRIX ADDEND                          INPUT
C        C      - NXM SUM OF A AND B                         OUTPUT
C        N      - NUMBER OF ROWS IN A, B, AND C              INPUT
C        M      - NUMBER OF COLUMNS IN A, B, AND C           INPUT
C
C***********************************************************************
C
      DIMENSION A(1), B(1), C(1)
C23456789
      IF ( (N .EQ. 0) .OR. (M .EQ. 0) ) RETURN
C
      NM = N * M
      DO 20 IJ = 1, NM
      C(IJ) = A(IJ) + B(IJ)
   20 CONTINUE
C23456789
      RETURN
      END
```

```
C***************************************************************
C    THIS ROUTINE COMPUTES THE INVERSE OF A MATRIX.
C    MATRIX INVERSION IS PERFORMED BY CROUT ELIMINATION.FOR A DESCRIPTION
C    OF THE METHOD,SEE HILDERBRAND,INTRODUCTION TO NUMERICAL ANALYSIS,
C    PAGE 429.
C
C    INPUT MATRIX :    [A]
C   OUTPUT MATRIX :    [AINV]
C           ORDER :       N
C      ERROR FLAG :     IER
C         [ADUM] IS A DUMMY MATRIX OF ORDER [N X N].
C
C
C                                          VISH (5-16-80)
C
C***************************************************************
      SUBROUTINE MATINV(A,AINV,N,ADUM,IER)
C***************************************************************
C
      IMPLICIT REAL*8 (A,R,D,E,S,T,P,C)
C
      DIMENSION A(N,N),AINV(N,N)
      DIMENSION ADUM(N,N)
C
      DATA EPS/1.0D-36/
C
C***************************************************************
C
C
C   'IER' IS A FLAG THAT IS SET TO '1' IF THE MATRIX IS SINGULAR.
      IER = 0
C
C**************************************** SPECIAL CASE FOR N=1
C
      IF(N.EQ.1)THEN
      IF(ABS(A(1,1))-EPS)1,1,2
   1  IER=1
      RETURN
   2  AINV(1,1) = 1.0/A(1,1)
      RETURN
      END IF
C23456789
```

```
C*******************************************************************
C
C
C     COPY [A] INTO [ADUM] BECAUSE [A] IS CHANGED DURING COMPUTATION.
C
      DO 6 I = 1,N
      DO 6 J = 1,N
    6 ADUM(I,J) = A(I,J)
C
C     INITIALIZE [AINV] TO [I].
C
      DO 20 I = 1,N
      DO 20 J = 1,N
      AINV(I,J) = 0.0D0
      IF(I.EQ.J) AINV(I,J) = 1.0D0
   20 CONTINUE
C
C
C
C     FIND THE LARGEST ELEMENT IN COL-1.
C
   30 IC = 0
      II = 0
      T  = ABS(ADUM(1,1))
      DO 40 I = 2,N
      IF(T-ABS(ADUM(I,1)) .GE. 0.0D0)GO TO 40
      II = I
      T  = ABS(ADUM(I,1))
   40 CONTINUE
C
C
      IF(II.EQ.0) GO TO 80
      IC = IC + 1
C
      DO 50 J = 1,N
      S = AINV(1,J)
      AINV(1,J) = AINV(II,J)
      AINV(II,J) = S
   50 CONTINUE
C
   60 DO 70 J = 1,N
      S = ADUM(1,J)
```

```
      ADUM(1,J) = ADUM(II,J)
      ADUM(II,J) = S
   70 CONTINUE
C
   80 P = ADUM(1,1)
      IF(ABS(P)-EPS) 90,90,100
C
C                                     DETERMINANT IS ZERO.
   90 CONTINUE
      D = 0.0
      IER = 1
      RETURN
C
C
C                  DETERMINANT IS NON-ZERO, CONTINUE
  100 DO 110 J = 2,N
      ADUM(1,J) = ADUM(1,J)/P
  110 CONTINUE
C
      DO 120 J = 1,N
      AINV(1,J) = AINV(1,J)/P
  120 CONTINUE
C
C
  130 DO 250 K = 2,N
      KM = K - 1
      T = -1.0
C23456789
      DO 150 I = K,N
      DP = ADUM(I,K)
C
      DO 140 J = 1,KM
      DP = DP - ADUM(I,J)*ADUM(J,K)
  140 CONTINUE
C
      ADUM(I,K) = DP
      IF(T-ABS(ADUM(I,K)) .GE. 0.0D0) GO TO 150
      T = ABS(ADUM(I,K))
      II = I
  150 CONTINUE
C
C
```

```
      IF(II-K .EQ. 0) GO TO 190
      IC = IC + 1
C
      DO 160  J = 1,N
      S = AINV(K,J)
      AINV(K,J) = AINV(II,J)
      AINV(II,J) = S
 160  CONTINUE
C
 170  DO 180 J = 1,N
      S = ADUM(K,J)
      ADUM(K,J) = ADUM(II,J)
      ADUM(II,J) = S
 180  CONTINUE
C
 190  DT = ADUM(K,K)
      IF(ABS(DT)-EPS .LE. 0.0D0) GO TO 90
      P = P*DT
      IF(K-N .EQ. 0) GO TO 220
      KP = K + 1
C
      DO 210 J = KP,N
      DP = ADUM(K,J)
C
      DO 200 I = 1,KM
      DP = DP - ADUM(K,I)*ADUM(I,J)
 200  CONTINUE
C
      ADUM(K,J) = DP/DT
 210  CONTINUE
 220  CONTINUE
C23456789
      DO 240 J = 1,N
      DP = AINV(K,J)
C
      DO 230 I = 1,KM
      DP = DP - ADUM(K,I)*AINV(I,J)
 230  CONTINUE
C
      AINV(K,J) = DP/DT
 240  CONTINUE
 250  CONTINUE
```

```
C
      IF(MOD(IC,2).EQ.0)GO TO 260
      P = -P
C
 260  D = P
      II = N
C
      DO 290 K = 2,N
      KP = II
      II = II - 1
C
      DO 280 J =1,N
      DP = AINV(II,J)
C
      DO 270 I = KP,N
      DP = DP - ADUM(II,I)*AINV(I,J)
 270  CONTINUE
C
      AINV(II,J) = DP
C
C
 280  CONTINUE
 290  CONTINUE
 300  CONTINUE
C
C
C
      RETURN
      END
```

```
      SUBROUTINE READ
      IMPLICIT REAL*8(A-H,O-Z)
C23456789
C     THIS SUBROUTINE READS INPUT DATA FROM
C          FILE UNIT 2
C
      COMMON /SYSTEM/ TIME,DELT,FIRST
C
      COMMON /TRUMOD/ TX(12),TXDOT(12),TXSIG(12),TXOM(12),TXOSIG(12),
     1               TMINV(6,6),TZSFE(12),TZBIAS(12)
C23456789
      COMMON /HMOD/ XHHAT(5),XHDOT(5),XHSIG(5),XHOM(5),XHOSIG(5),
     1             HMINV(5,5),AH(5,5),BH(5),PH(5,5),QH(5,5),
     1             RHSIG(5),RH(5,5),HH(5,5)
C
      COMMON /VMOD/ XVHAT(4),XVDOT(4),XVSIG(4),XVOM(4),XVOSIG(4),
     1             VMINV(4,4),AV(4,4),BV(4),PV(4,4),QV(4,4),
     1             RVSIG(4),RV(4,4),HV(4,4)
C
C     READ PROCESS NOISE, INITIAL STATE, INITIAL STATE ERROR,
C     MEASUREMENT SCALE FACTOR ERROR, MEASUREMENT BIAS ERROR FOR
C     THE TRUTH MODEL
C
      READ(2,*) TXSIG,TXOM,TXOSIG,TZSFE,TZBIAS
C
C     READ PROCESS NOISE, INITIAL STATE, INITIAL STATE ERROR,
C     MEASUREMENT ERROR FOR THE HORIZONTAL FILTER MODEL
C
      READ(2,*) XHSIG,XHOM,XHOSIG,RHSIG
C
C     READ PROCESS NOISE, INITIAL STATE, INITIAL STATE ERROR,
C     MEASUREMENT ERROR FOR THE VERTICAL FILTER MODEL.
C
      READ(2,*) XVSIG,XVOM,XVOSIG,RVSIG
C
      RETURN
      END
```

```
      SUBROUTINE TMTRIX(A,C)
      IMPLICIT REAL*8(A-H,O-Z)
C
C     THIS SUBROUTINE GENERATES TRANSFORMATION MATRIX C(3,3)
C     FOR A GIVEN EULER ANGLES A(3)
C
      DIMENSION A(3),C(3,3)
C
      P=A(1)
      T=A(2)
      S=A(3)
C
      SP=DSIN(P)
      CP=DCOS(P)
      ST=DSIN(T)
      CT=DCOS(T)
      SS=DSIN(S)
      CS=DCOS(S)
C23456789
      C(1,1)=CT*CS
      C(1,2)=SP*ST*CS-CP*SS
      C(1,3)=CP*ST*CS+SP*SS
      C(2,1)=CT*SS
      C(2,2)=SP*ST*SS+CP*CS
      C(2,3)=CP*ST*SS-SP*CS
      C(3,1)=-ST
      C(3,2)=SP*CT
      C(3,3)=CP*CT
C
      RETURN
      END
```

```
      FUNCTION GAUSS ( SIGMA )
        IMPLICIT REAL*8(A-H,O-Z)
C
C***********************************************************************
C
C     PROGRAM NAME - AERODYNAMIC COEFFICIENT ESTIMATION (ACES)
C
C     ROUTINE NAME - GAUSS                         PROG. NO. XXX
C
C***********************************************************************
C
C     FUNCTION GAUSS ( SIGMA )
C
C     PURPOSE
C        COMPUTE A RANDOM NUMBER FROM A GAUSSIAN DISTRIBUTION WITH
C        A MEAN OF ZERO AND A STANDARD DEVIATION OF SIGMA.
C
C     ARGUMENTS                                            INPUT/OUTPUT
C        SIGMA  - STANDARD DEVIATION                           INPUT
C
C***********************************************************************
C23456789
      DIMENSION TABLE(258), ATABLE(91), BTABLE(90), CTABLE(77)
      EQUIVALENCE (TABLE(1),  ATABLE(1)),
     A            (TABLE(92), BTABLE(1)),
     B            (TABLE(182),CTABLE(1))
      DATA ATABLE  /0.0000000000,
     A 0.0048957779,0.0097916732,0.0146878031,0.0195842852,0.0244812369,
     B 0.0293787757,0.0342770194,0.0391760855,0.0440760921,0.0489771572,
     C 0.0538793990,0.0587829361,0.0636878869,0.0685943705,0.0735025060,
     D 0.0784124127,0.0833242105,0.0882380194,0.0931539598,0.0980721525,
     E 0.1029927185,0.1079157795,0.1128414574,0.1177698746,0.1227011540,
     F 0.1276354191,0.1325727936,0.1375134021,0.1424573696,0.1474048216,
     G 0.1523558843,0.1573106846,0.1622693499,0.1672320084,0.1721987889,
     H 0.1771698210,0.1821452351,0.1871251622,0.1921097344,0.1970990843,
     I 0.2020933456,0.2070926527,0.2120971412,0.2171069472,0.2221222082,
     J 0.2271430625,0.2321696494,0.2372021093,0.2422405838,0.2472852153,
     K 0.2523361478,0.2573935261,0.2624574964,0.2675282061,0.2726058039,
     L 0.2776904398,0.2827822652,0.2878814328,0.2929880969,0.2981024129,
     M 0.3032245382,0.3083546313,0.3134928526,0.3186393640,0.3237943289,
     N 0.3289579126,0.3341302823,0.3393116065,0.3445020561,0.3497018036,
     O 0.3549110233,0.3601298918,0.3653585875,0.3705972911,0.3758461852,
```

```
     P 0.3811054548,0.3863752869,0.3916558711,0.3969473992,0.4022500653,
     Q 0.4075640663,0.4128896014,0.4182268725,0.4235760842,0.4289374437,
     R 0.4343111612,0.4396974496,0.4450965250,0.4505086063,0.4559339156/
      DATA BTABLE
     A/0.4613726783,0.4668251229,0.4722914813,0.4777719889,0.4832668847,
     B 0.4887764111,0.4943008145,0.4998403449,0.5053952563,0.5109658067,
     C 0.5165522584,0.5221548776,0.5277739351,0.5334097062,0.5390624707,
     D 0.5447325130,0.5504201225,0.5561255936,0.5618492257,0.5675913235,
     E 0.5733521972,0.5791321623,0.5849315401,0.5907506581,0.5965898493,
     F 0.6024494532,0.6083298156,0.6142312891,0.6201542326,0.6260990123,
     G 0.6320660016,0.6380555809,0.6440681386,0.6501040706,0.6561637812,
     H 0.6622476825,0.6683561956,0.6744897502,0.6806487851,0.6868337486,
     I 0.6930450984,0.6992833024,0.7055488387,0.7118421959,0.7181638739,
     J 0.7245143835,0.7308942474,0.7373040004,0.7437441897,0.7502153755,
     K 0.7567181311,0.7632530437,0.7698207150,0.7764217611,0.7830568137,
     L 0.7897265199,0.7964315437,0.8031725656,0.8099502840,0.8167654153,
     M 0.8236186951,0.8305108782,0.8374427401,0.8444150774,0.8514287083,
     N 0.8584844741,0.8655832398,0.8727258946,0.8799133538,0.8871465590,
     O 0.8944264796,0.9017541138,0.9091304899,0.9165566675,0.9240337388,
     P 0.9315628300,0.9391451029,0.9467817563,0.9544740278,0.9622231953,
     Q 0.9700305792,0.9778975439,0.9858255004,0.9938159079,1.0018702764,
     R 1.0099901692,1.0181772056,1.0264330631,1.0347594811,1.0431582633/
      DATA CTABLE
     A/1.0516312817,1.0601804794,1.0688078753,1.0775155670,1.0863057363,
     B 1.0951806528,1.1041426793,1.1131942772,1.1223380117,1.1315765584,
     C 1.1409127093,1.1503493804,1.1598896185,1.1695366102,1.1792936900,
     D 1.1891643502,1.1991522510,1.2092612317,1.2194953228,1.2298587592,
     E 1.2403559942,1.2509917155,1.2617708616,1.2726986412,1.2837805526,
     F 1.2950224067,1.3064303511,1.3180108973,1.3297709502,1.3417178411,
     G 1.3538593641,1.3662038164,1.3787600432,1.3915374880,1.4045462482,
     H 1.4177971380,1.4313017591,1.4450725798,1.4591230250,1.4734675779,
     I 1.4881218960,1.5031029431,1.5184291412,1.5341205444,1.5501990408,
     J 1.5666885861,1.5836154758,1.6010086649,1.6189001435,1.6373253828,
     K 1.6563238653,1.6759397228,1.6962225050,1.7172281175,1.7390199717,
     L 1.7616704104,1.7852624904,1.8098922385,1.8356715369,1.8627318674,
     M 1.8912292378,1.9213507743,1.9533237077,1.9874278859,2.0240136237,
     N 2.0635278983,2.1065540882,2.1538746941,2.2065752165,2.2662268092,
     O 2.3352330401,2.4175590162,2.5205022172,2.6600674686,2.8856349124,
     P 3.2500000000,3.2500000000/
       ISIGMA=SIGMA
C23456789
      X = RAN(ISIGMA)
```

```
C
C    IF SIGMA IS NONPOSITIVE, SET GAUSS TO ZERO AND RETURN
C    WHILE PRESERVING THE SEQUENCE OF RANDOM NUMBERS GENERATED
C
      IF (SIGMA .LE. 0.0) GO TO 20
         XI = 512.0*X
         I = XI
C23456789
C    BRANCH TO ALGORITHM FOR TABLE LOOK-UP AND INTERPOLATION
C
         IF (XI .GT. 256.0) GO TO 10
            Y = (XI - I)*(TABLE(I+2) - TABLE(I+1)) + TABLE(I+1)
            GAUSS = Y*SIGMA
            GO TO 30
C
  10     Y = (XI - I)*(TABLE(I-255) - TABLE(I-254)) - TABLE(I-255)
         GAUSS = Y*SIGMA
         GO TO 30
C
  20  GAUSS = 0.0
  30  CONTINUE
C
      RETURN
      END
```